Part III: Gear Systems: Analysis

This section will review standard gear systems and will provide the basic tools to perform analysis on these systems. The areas covered in this section are:

1) Brief history of gears
2) Gears 101: The geometric details about standard gears (involute)
3) Salient features of involute gears
4) Gear-tooth geometry equations
5) Gear train systems: fixed-axis and Planetary
6) Types of Gears

Brief History:

27th Century BC: Earliest recorded use of gears on Chinese "South pointing Chariot" (also first use of a differential)
4th Century BC: Aristotle gave first known written description of gears
1st Century BC: Antikythera mechanism – geared mechanism to predict the locations of stars and planets in the sky. (A planetary gear train mechanism)
Prior to 1600's: Gears did not exist in modern form
1600's: Christian Huygens proposed the involute profile
1600's: International Challenge to build an accurate clock for sea travel led to the development of the wrist watch, and the basis for modern gears)
1700's: Leonhard Euler proposed the use of involutes for gear teeth
1835: Hobbing process patented

Gears 101: Fundamental Law of Gearing

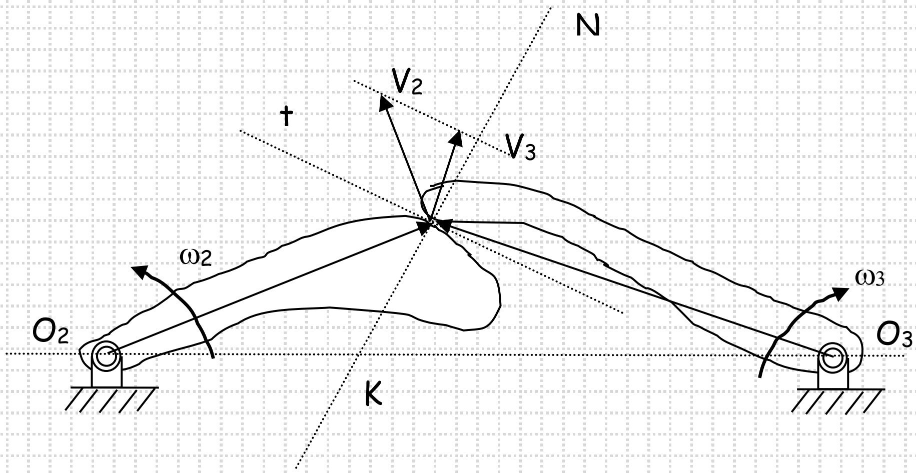
"Angular velocity ratio between gears must remain constant throughout their mesh"

<u>Why?</u>

Constant velocity transmission ➔ no accelerations generated in transfer of motion (i.e., no forces)
No speed or torque variations

Kinematic requirement for constant velocity:

Consider two bodies connected with a higher pair joint. For constant velocity, the line of action and line of centers must intersect at a constant location:



Notes:

1) A common Normal and tangent (N, t) exist and are defined by the 2 surfaces
2) "Condition of contact": no relative motion can occur along the common normal
3) All sliding takes place along the common tangent
4) The result of these rules (plus some geometric construction not shown here):

$$\frac{\omega_3}{\omega_2} = \frac{O_2K}{O_3K}$$

5) Requirement for constant velocity:

6) Requirement for no sliding:

In summary:

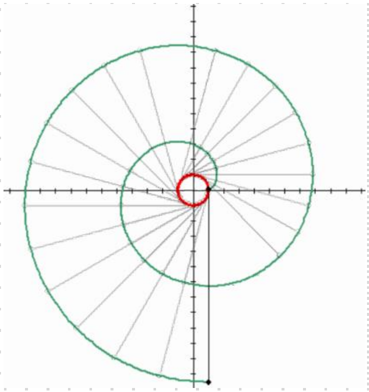
Fundamental law of gearing ➔

Rules for all Gears:

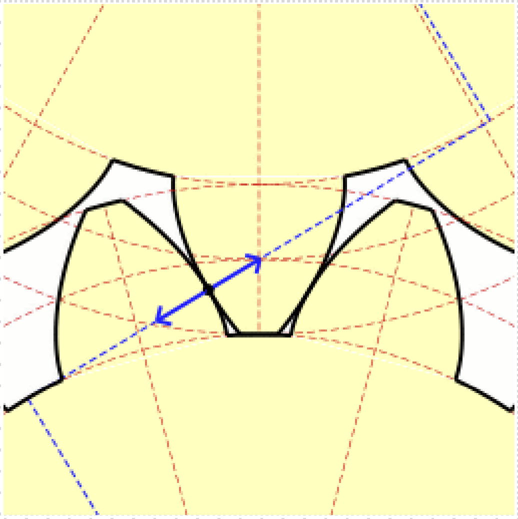
$$GR = m_v = \frac{\omega_o}{\omega_i} = \pm\frac{r_i}{r_o} = \pm\frac{N_i}{N_o}$$

Gears 102: Details about the involute gear profile:
A number options are available to satisfy the fundamental law of gearing, but the involute is preferred because of many special properties (to be discussed).

1) Involute of a circle:



2) Consider two bodies to be connected with gears. Do the following:
1) First, represent these as base circles (the base of the teeth).
2) Create a line connecting their centers, this is constant.
3) create a line that will intersect the line of centers at a constant point. Choose a common tangent between the circles to do this. (note, as bodies rotate, this line is constant)
4) note that if this line is the common normal of the gear teeth, than it would satisfy the law of gearing.
5) Define your gear teeth such that their surface is perpendicular to this line (as they rotate).
6) Rotate the base circle to generate the tooth surface as an Involute profile.

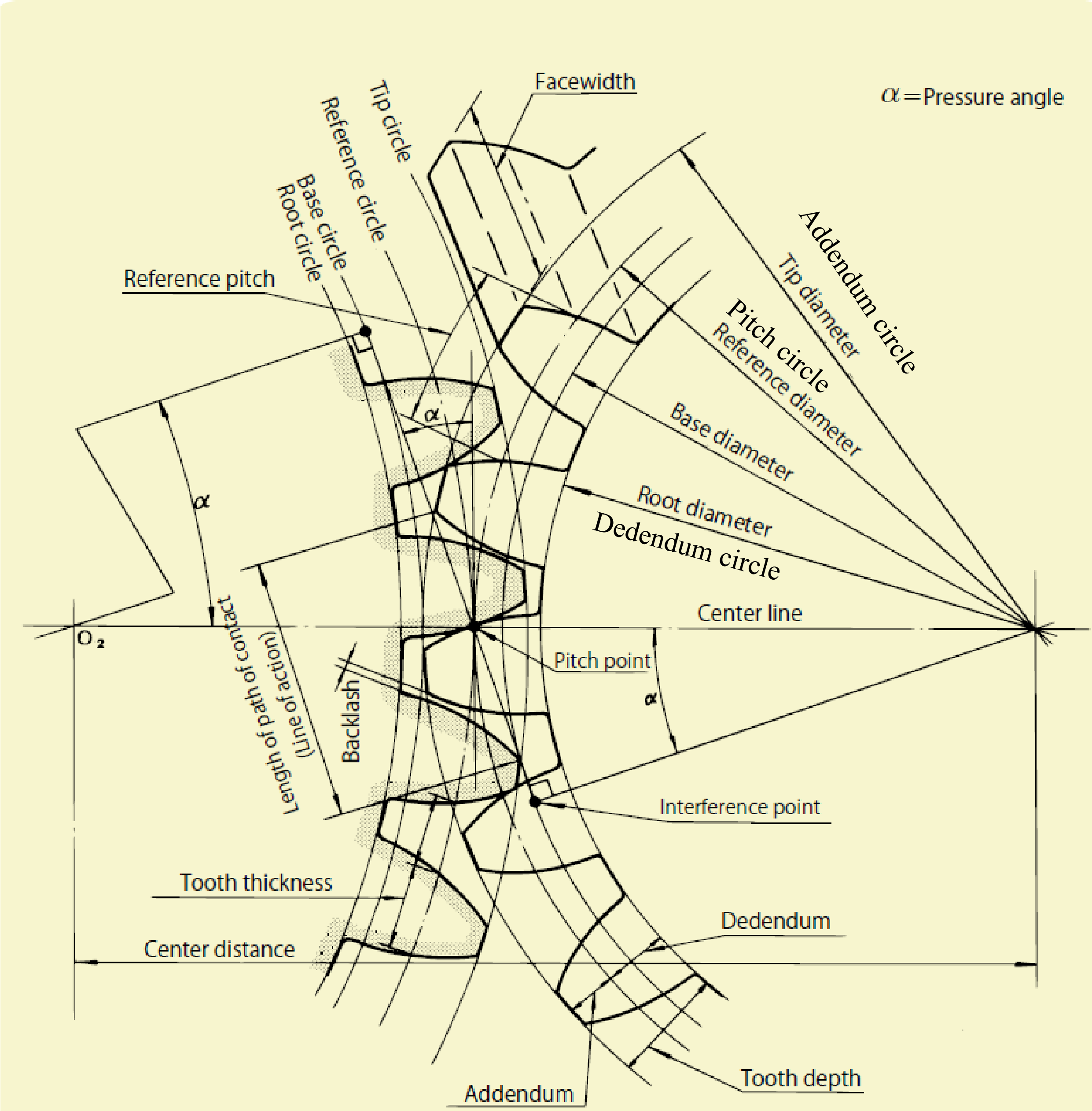


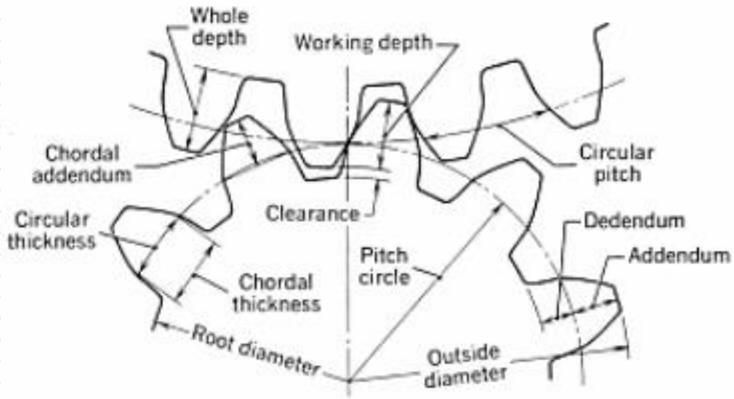
See Wikipedia, involute gear for animation

3) <u>A primary advantage of the Involute profile:</u>

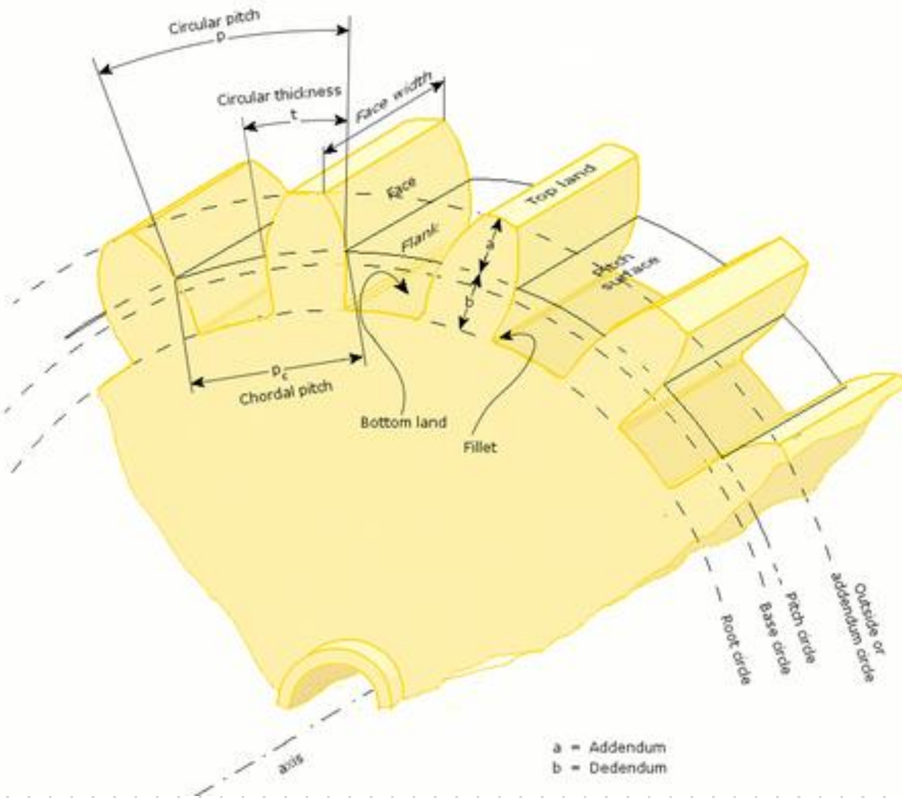
1. Velocity ratio of gears in mesh is constant
2. Velocity ratio is independent of center distance (just like belts and pulleys).
3. Line of Action: tangent to the base circles, all tooth contact takes place on this line.
4. Involutes conjugate to themselves

Gears 103: Gear tooth Terminology:
Based on this involute geometry of gear teeth, the geometry of a gear can be standardized and named, as in the following figures.

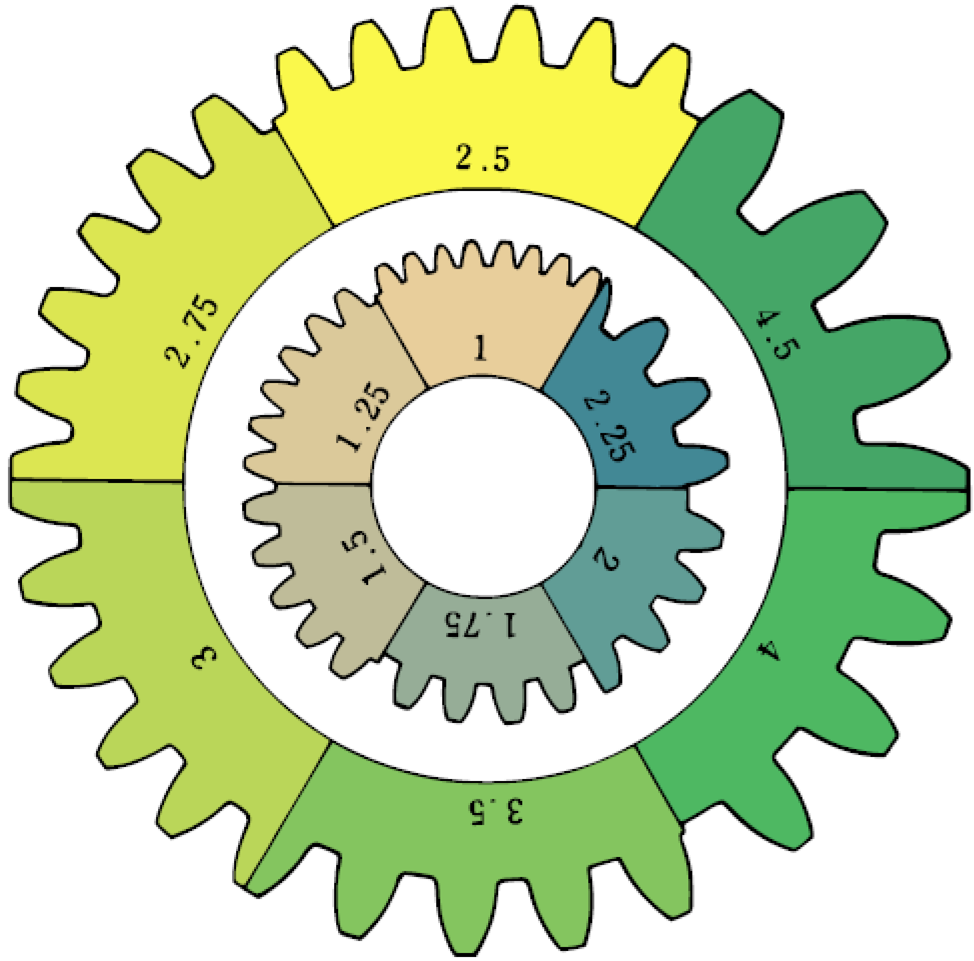




Spur gear terms.



<u>Gear Pitch:</u>
Pitch = Diametral Pitch = $P = P_d = N/D$
(See figure in Norton).

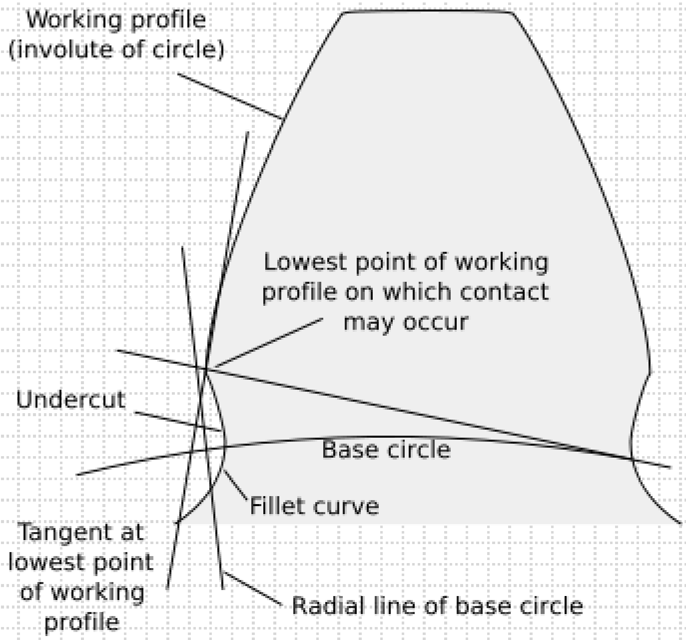


From khk gears (note, in module)

So gear teeth are involutes, is the entire surface of the gear tooth an involute?
Three cases:
1) all of the gear tooth is an involute (if $R_d > R_b$)

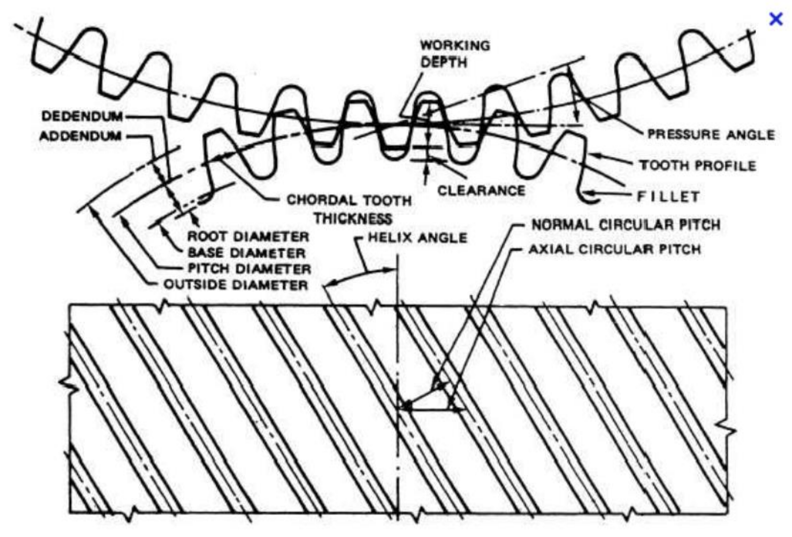
2) part of the gear tooth is involute, part is a straight line ($R_b > R_d$)

3) part of the gear tooth is involute, and part is undercut (see figure below).



Why is a gear tooth undercut?

Involutes apply to all gear types:
Consider helical gear



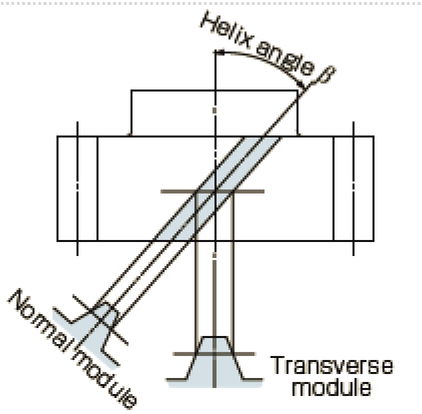


Fig. 2.9 Right-handed Helical Gear

From khkgears.co.jp

Conditions of Interchangeability (For Standard Gears)

1.

2.

3.

Details of Involute Gears

Spur Gear Formulas

To Find	$14\frac{1}{2}$ degree Pressure Angle	20 and 25 degree Pressure Angles
Addendum, a	$a = \frac{1.0}{P}$	$a = \frac{1.0}{P}$
Dedendum, b	$b = \frac{1.157}{P}$	$b = \frac{1.250}{P}$
Pitch diameter, D	$D = \frac{N}{P}$	$D = \frac{N}{P}$
Outside diameter, D_o	$D_o = \frac{N + 2}{P}$	$D_o = \frac{N + 2}{P}$
Number of teeth, N	$N = D \times P$	$N = D \times P$
Tooth thickness, t	$t = \frac{1.5708}{P}$	$t = \frac{1.5708}{P}$
Whole depth, h_t	$h_t = \frac{2.157}{P}$	$h_t = \frac{2.250}{P}$
Clearance, c	$c = \frac{.157}{P}$	$c = \frac{.250}{P}$
Center distance, C	$C = \frac{N_1 + N_2}{2 \times P}$	$C = \frac{N_1 + N_2}{2 \times P}$
Working depth, h_k	$h_k = \frac{2}{P}$	$h_k = \frac{2}{P}$
Chordal tooth thickness, t_c	$t_c = D \sin\left(\frac{90 \text{ degrees}}{N}\right)$	$t_c = D \sin\left(\frac{90 \text{ degrees}}{N}\right)$
Chordal addendum, a_c	$a_c = a + \frac{t^2}{4D}$	$a_c = a + \frac{t^2}{4D}$
Diametral pitch, P	$P = \frac{N}{D}$	$P = \frac{N}{D}$
Center distance, C	$C = \frac{D_1 + D_2}{2}$	$C = \frac{D_1 + D_2}{2}$

Salient Features of Involute Gears:

5. Velocity ratio of gears in mesh is constant and independent of center distance (just like belts and pulleys).
6. Line of Action: tangent to the base circles, all tooth contact takes place on this line.
7. Interference points: points where line of action meets the base circles.
8. If contact occurs outside the interference points, interference will occur between the gears.
9. A gear that can mesh with a rack with out undercutting can mesh with any other gear of equal or greater size without undercutting.
10. Standard gears with the same diametral pitch and same pressure angle are interchangeable.
11. Contact ratio, mp, is the average number of teeth in contact (in design, 1.2 is a minimum, the larger the better).
12. A rack is a gear with infinite diameter. On a rack, the involute becomes a straight line.
13. Common gear cutters: Hobs, and Shapers. Hobs act like a bit in a milling machine. Shapers have a translating cutting stroke.
14. The type of cutter determines the minimum number of teeth possible with out undercutting. The hob represents the worst case, since it is like a rack. For a gear cut with a shaper, the minimum number of teeth may be given by the shaper, if N of the shaper is known.
15. Salient: (sa' li-ent) *adj.* Standing out from the rest; noticeable; conspicuous; prominent. (Webster's, College Ed.)

Details of Involute Gears

1. Equations not found in Norton:
(Refer to Fig. 1)

Base pitch (distance between one tooth set measured along base circle):

$$p_b = \frac{2\pi R_b}{N} \qquad (1)$$

Length of action:

$$Z = \sqrt{R_{o1}^2 - R_{b1}^2} + \sqrt{R_{o2}^2 - R_{b2}^2} - C\sin(\phi) \qquad (2)$$

Contact ratio (average number of teeth in contact):

$$m_p = \frac{Z}{p_b} \qquad (3)$$

Diametral Pitch (number of teeth per inch):

$$P_d = \frac{N}{D} \qquad (4)$$

Module (mm per tooth):

$$m = \frac{D(mm)}{N} \qquad (5)$$

Minimum number of teeth to avoid interference: (*k*=1 for full depth teeth)

a) for a rack:

$$N = \frac{2k}{\sin^2(\phi)} \qquad (6)$$

b) for two gears in mesh:

$$N_1^2 + 2N_2 N_1 - \frac{4k}{\sin^2(\phi)}(N_2 + k) = 0 \qquad (7)$$

Center distance:

$$C = R_1 + R_2 = \frac{N_1 + N_2}{2P_d} \qquad (8)$$

"operating" center distance and pressure angle:

$$C'\cos(\phi') = C\cos(\phi) \qquad (9)$$

Backlash resulting from an increased operating center distance:

$$B = 2C'\left(inv(\phi') - inv(\phi)\right), \quad inv(\phi) = \tan(\phi) - \phi \qquad (10)$$

Tooth thickness: (requires the tooth thickness at some radius to be known, generally at the pitch circle):

$$t_B = 2R_B\left[\frac{t_A}{2R_A} + inv(\phi_A) - inv(\phi_B)\right] \qquad (11)$$

Radius and angle at various points along the involute:

$$R_b = R_A\cos(\phi_A) = R_B\cos(\phi_B) = R\cos(\phi) \qquad (12)$$

Concept questions:

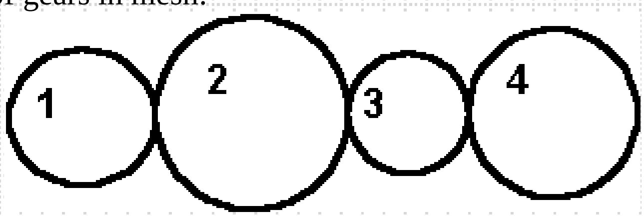
1. 2 gears are in mesh, 20deg. Pressure angle, pitch = 6, N1 = 24, N2 = 48, transmitting a torque. What is the ratio of the radial force on the bearings to the tangential force between the gears.

Gear-Train Systems:
Gears are used in combinations to create a desired torque/velocity ratio. Combinations of gears can be divided into two classes: Fixed-axis gear trains, and planetary gear trains.

<u>Fixed-axis gear trains:</u>

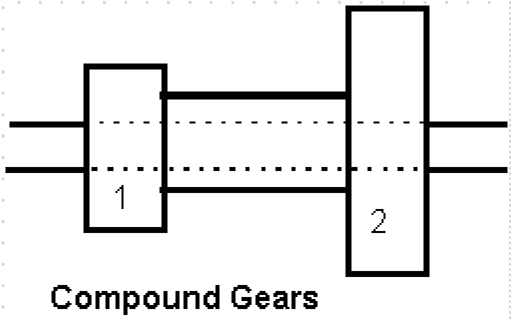
The sign change occurs for external gears. The diameters listed are the pitch diameters.

Now consider a series of gears in mesh:

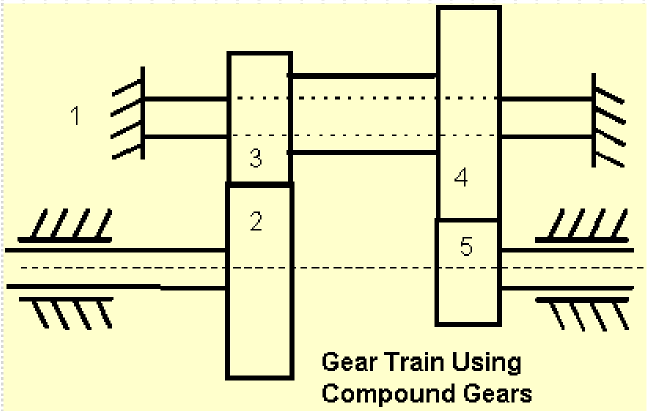


In this arrangement, the intermediate gears do not affect the overall velocity ratio, and therefore should be replaced with a more cost effective means of power transmission. Only the outer two gears are useful in achieving the desired velocity ratio. Since the velocity ratio of a single gear set is practically limited to 10:1 or less (actually, more like 5:1 greater), compound gears (two gears constrained to have the same angular velocity) are used in gears trains to achieve larger velocity ratios:

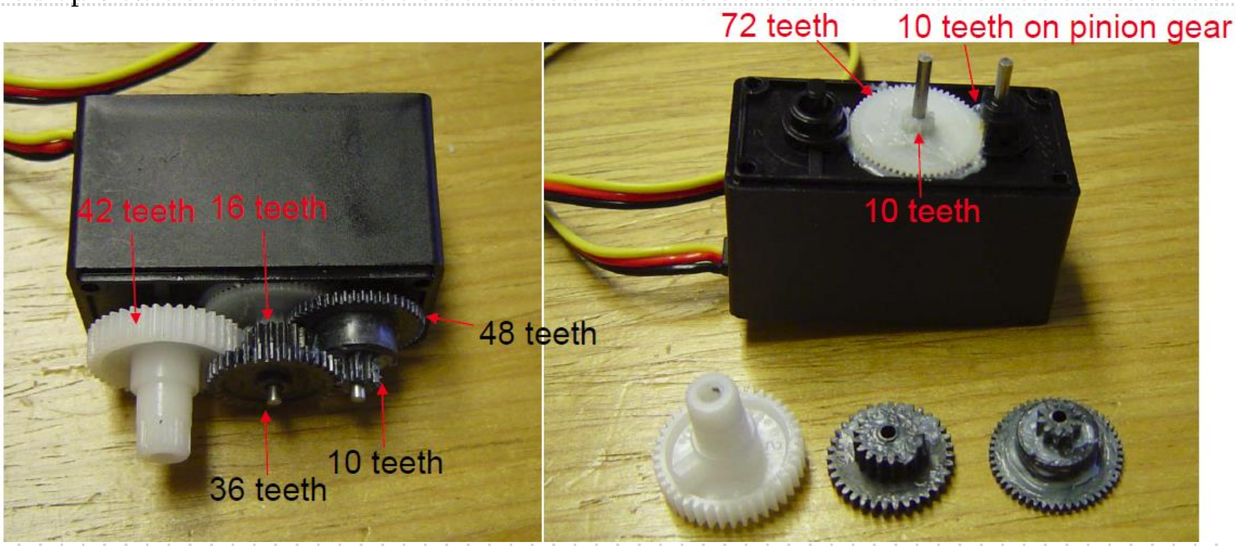
Compound Gears:

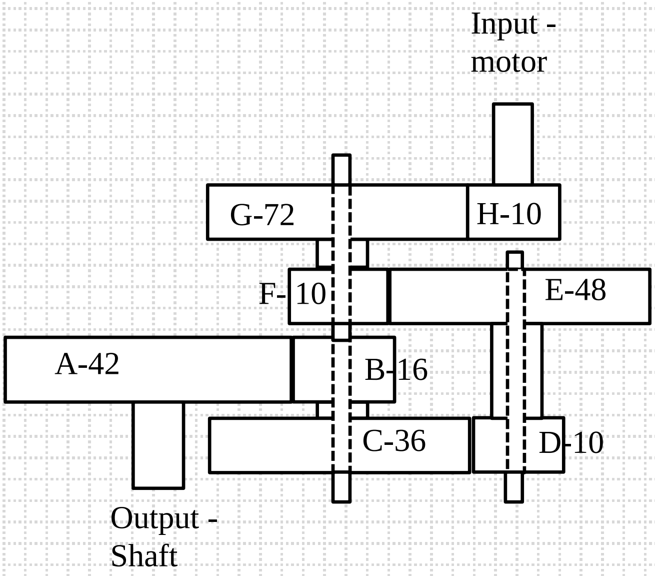


Example 1:



Example 2:


72 teeth
10 teeth on pinion gear
10 teeth
42 teeth
16 teeth
48 teeth
36 teeth
10 teeth


Input -
motor
G-72
H-10
F-10
E-48
A-42
B-16
C-36
D-10
Output -
Shaft

Gear train design:

Gear train synthesis or design is the process of selecting the design parameters in a gear train system to meet desired objectives. A portion of these are basic criteria for the design of gear-trains, and a portion of these objectives can be stated as constraints that exist as functions of the design parameters. First, the basic criteria:

1) Gears in mesh must have the same p_d, ϕ
2) Generally, all gears within a gear train will have the same p_d, ϕ (but this is preference only, not a requirement.
3) All gear teeth must be integer values, greater than a min. value, (perhaps stock size)

Next, the functional requirements:

1) Gear ratio
2) center distance requirements on gears
3) reverted gear train
4) relative direction of input/output

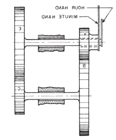
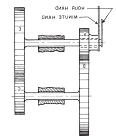
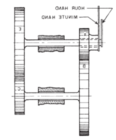
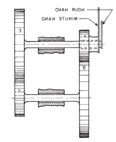
Design procedure

1) Select the number of stages

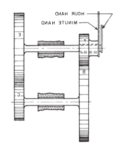
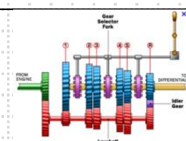
2) List the functional requirements as equations. Combine *n* equations to eliminate *n* design parameters

3) Solve for remaining design parameters

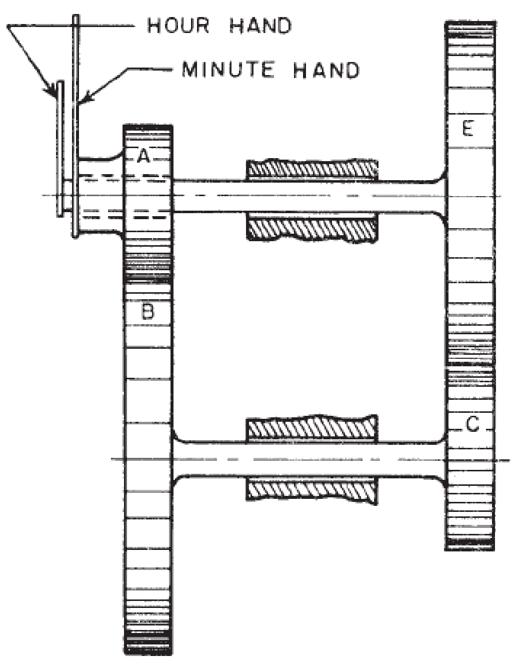
Functional requirements on a Gear train:

#	Requirement	Example	reference
1	Gear ratio	$m_v = \dfrac{N_3 N_5}{N_2 N_4}$	2-stage train 
2	Center distance	$C_1 = R_2 + R_3 = \dfrac{N_2 + N_3}{2p_d}$	
3	Center distance	$C_2 = R_4 + R_5 = \dfrac{N_4 + N_5}{2p_d}$	
4	Reverted (equal center distance)	$C_1 = C_2$ $\dfrac{N_2 + N_3}{2p_d} = \dfrac{N_4 + N_5}{2p_d}$ $N_2 + N_3 = N_4 + N_5$	

Functional requirements on a Gear train:

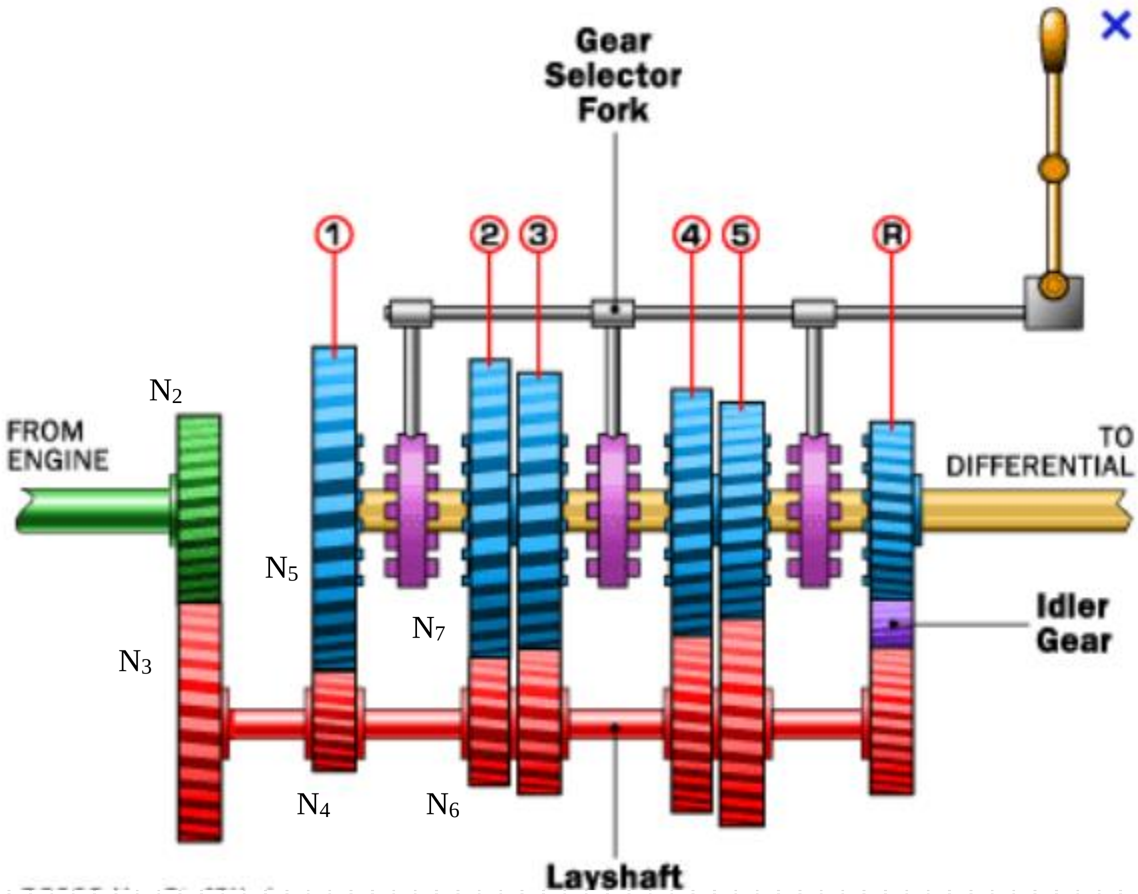
#	Type	Design parameters	reference
1	2 stage	4 $N_2, N_3 N_4, N_5$	
2	3 stage	6	
	8 stage		

Example: Design a gear mechanism to drive the hour/minute hands of a clock. Assume the minute hand is input, hour hand is output and input/output are inline. Select gears assuming p_d is 24 and find nominal center distance.



	Gear train Design spreadsheet														
	Spreadsheet for 4-gear 2-stage reverted train						Gear Ratic	60							
NC (down	NE (across)														
		Table of NA													
	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116
12	5.538462	6.204082	6.903553	7.636364	8.40201	9.2	10.02985	10.89109	11.78325	12.70588	13.65854	14.64078	15.65217	16.69231	17.76077
13	5.214286	5.838863	6.495283	7.183099	7.901869	8.651163	9.430556	10.23963	11.07798	11.94521	12.84091	13.76471	14.71622	15.69507	16.70089
14	4.933333	5.522124	6.140969	6.789474	7.467249	8.173913	8.909091	9.672414	10.46352	11.28205	12.12766	13	13.89873	14.82353	15.77406
15	4.6875	5.244813	5.830579	6.444444	7.086066	7.755102	8.45122	9.174089	9.923387	10.6988	11.5	12.32669	13.17857	14.05534	14.95669
16	4.470588	5	5.55642	6.139535	6.749035	7.384615	8.045977	8.732824	9.444867	10.18182	10.9434	11.72932	12.53933	13.37313	14.23048
17	4.277778	4.782288	5.3125	5.868132	6.448905	7.054545	7.684783	8.33935	9.017986	9.72043	10.44643	11.19573	11.96809	12.76325	13.58099
18	4.105263	4.587413	5.094077	5.625	6.179931	6.758621	7.360825	7.986301	8.634812	9.306122	10	10.71622	11.45455	12.21477	12.99666
19	3.95	4.41196	4.897351	5.405941	5.9375	6.491803	7.068627	7.667752	8.288961	8.932039	9.596774	10.28296	10.99038	11.71885	12.46815
20	3.809524	4.253165	4.719243	5.207547	5.717868	6.25	6.803738	7.378882	7.975232	8.592593	9.230769	9.889571	10.56881	11.26829	11.98784
21	3.681818	4.108761	4.557229	5.027027	5.517964	6.029851	6.5625	7.115727	7.689349	8.283186	8.897059	9.530792	10.18421	10.85714	11.54942
22	3.565217	3.976879	4.409222	4.862069	5.335244	5.828571	6.34188	6.875	7.427762	8	8.591549	9.202247	9.831933	10.48045	11.14763
23	3.458333	3.855956	4.273481	4.710744	5.167582	5.643836	6.139344	6.653951	7.1875	7.739837	8.310811	8.90027	9.508065	10.13405	10.77807
24	3.36	3.744681	4.148541	4.571429	5.013193	5.473684	5.952756	6.450262	6.966057	7.5	8.051948	8.621762	9.209302	9.814433	10.43702
25	3.269231	3.641944	4.033163	4.442748	4.870558	5.316456	5.780303	6.261965	6.761307	7.278195	7.8125	8.36409	8.932836	9.51861	10.12129
26	3.185185	3.546798	3.92629	4.323529	4.738386	5.170732	5.620438	6.087379	6.571429	7.072464	7.590361	8.125	8.676259	9.244019	9.828162
		table of NB													
	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116
12	66.46154	69.79592	73.09645	76.36364	79.59799	82.8	85.97015	89.10891	92.21675	95.29412	98.34146	101.3592	104.3478	107.3077	110.2392
13	67.78571	71.16114	74.50472	77.8169	81.09813	84.34884	87.56944	90.76037	93.92202	97.05479	100.1591	103.2353	106.2838	109.3049	112.2991
14	69.06667	72.47788	75.85903	79.21053	82.53275	85.82609	89.09091	92.32759	95.53648	98.71795	101.8723	105	108.1013	111.1765	114.2259
15	70.3125	73.75519	77.16942	80.55556	83.91393	87.2449	90.54878	93.82591	97.07661	100.3012	103.5	106.6733	109.8214	112.9447	116.0433
16	71.52941	75	78.44358	81.86047	85.25097	88.61538	91.95402	95.26718	98.55513	101.8182	105.0566	108.2707	111.4607	114.6269	117.7695
17	72.72222	76.21771	79.6875	83.13187	86.55109	89.94545	93.31522	96.66065	99.98201	103.2796	106.5536	109.8043	113.0319	116.2367	119.419
18	73.89474	77.41259	80.90592	84.375	87.82007	91.24138	94.63918	98.0137	101.3652	104.6939	108	111.2838	114.5455	117.7852	121.0033
19	75.05	78.58804	82.10265	85.59406	89.0625	92.5082	95.93137	99.33225	102.711	106.068	109.4032	112.717	116.0096	119.2812	122.5318
20	76.19048	79.74684	83.28076	86.79245	90.28213	93.75	97.19626	100.6211	104.0248	107.4074	110.7692	114.1104	117.4312	120.7317	124.0122
21	77.31818	80.89124	84.44277	87.97297	91.48204	94.97015	98.4375	101.8843	105.3107	108.7168	112.1029	115.4692	118.8158	122.1429	125.4506
22	78.43478	82.02312	85.59078	89.13793	92.66476	96.17143	99.65812	103.125	106.5722	110	113.4085	116.7978	120.1681	123.5196	126.8524
23	79.54167	83.14404	86.72652	90.28926	93.83242	97.35616	100.8607	104.346	107.8125	111.2602	114.6892	118.0997	121.4919	124.866	128.2219
24	80.64	84.25532	87.85146	91.42857	94.98681	98.52632	102.0472	105.5497	109.0339	112.5	115.9481	119.3782	122.7907	126.1856	129.563
25	81.73077	85.35806	88.96684	92.55725	96.12944	99.68354	103.2197	106.738	110.2387	113.7218	117.1875	120.6359	124.0672	127.4814	130.8787
26	82.81481	86.4532	90.07371	93.67647	97.26161	100.8293	104.3796	107.9126	111.4286	114.9275	118.4096	121.875	125.3237	128.756	132.1718

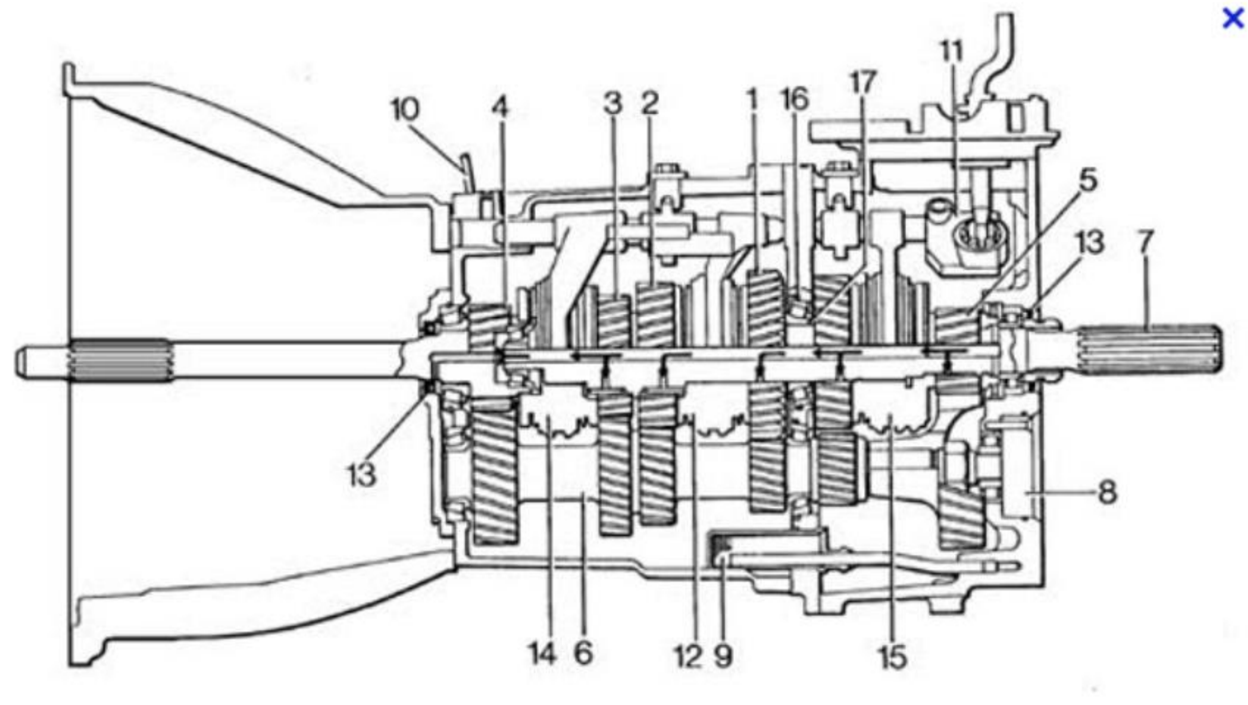
	Gear train Design spreadsheet														
	Spreadsheet for 4-gear 2-stage reverted train						Gear Ratic	12							
NC (down	NE (across)														
		Table of NA													
	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116
12	21.17647	23.38462	25.66038	28	30.4	32.85714	35.36842	37.93103	40.54237	43.2	45.90164	48.64516	51.42857	54.25	57.10769
13	20.27778	22.4	24.58929	26.84211	29.15517	31.52542	33.95	36.42623	38.95161	41.52381	44.14063	46.8	49.5	52.23881	55.01471
14	19.47368	21.51724	23.62712	25.8	28.03279	30.32258	32.66667	35.0625	37.50769	40	42.53731	45.11765	47.73913	50.4	53.09859
15	18.75	20.72131	22.75806	24.85714	27.01563	29.23077	31.5	33.8209	36.19118	38.6087	41.07143	43.57746	46.125	48.71233	51.33784
16	18.09524	20	21.96923	24	26.08955	28.23529	30.43478	32.68571	34.98592	37.33333	39.72603	42.16216	44.64	47.15789	49.71429
17	17.5	19.34328	21.25	23.21739	25.24286	27.32394	29.45833	31.64384	33.87838	36.16	38.48684	40.85714	43.26923	45.72152	48.2125
18	16.95652	18.74286	20.59155	22.5	24.46575	26.48649	28.56	30.68421	32.85714	35.07692	37.34177	39.65	42	44.39024	46.81928
19	16.45833	18.19178	19.98649	21.84	23.75	25.71429	27.73077	29.79747	31.9125	34.07407	36.28049	38.53012	40.82143	43.15294	45.52326
20	16	17.68421	19.42857	21.23077	23.08861	25	26.96296	28.97561	31.03614	33.14286	35.29412	37.48837	39.72414	42	44.31461
21	15.57692	17.21519	18.9125	20.66667	22.47561	24.33735	26.25	28.21176	30.22093	32.27586	34.375	36.51685	38.7	40.92308	43.18478
22	15.18519	16.78049	18.43373	20.14286	21.90588	23.72093	25.58621	27.5	29.46067	31.46667	33.51648	35.6087	37.74194	39.91489	42.12632
23	14.82143	16.37647	17.98837	19.65517	21.375	23.14607	24.96667	26.83516	28.75	30.70968	32.71277	34.75789	36.84375	38.96907	41.13265
24	14.48276	16	17.57303	19.2	20.87912	22.6087	24.3871	26.21277	28.08421	30	31.95876	33.95918	36	38.08	40.19802
25	14.16667	15.64835	17.18478	18.77419	20.41489	22.10526	23.84375	25.62887	27.45918	29.33333	31.25	33.20792	35.20588	37.24272	39.31731
26	13.87097	15.31915	16.82105	18.375	19.97938	21.63265	23.33333	25.08	26.87129	28.70588	30.58252	32.5	34.45714	36.45283	38.48598
		table of NB													
	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116
12	50.82353	52.61538	54.33962	56	57.6	59.14286	60.63158	62.06897	63.45763	64.8	66.09836	67.35484	68.57143	69.75	70.89231
13	52.72222	54.6	56.41071	58.15789	59.84483	61.47458	63.05	64.57377	66.04839	67.47619	68.85938	70.2	71.5	72.76119	73.98529
14	54.52632	56.48276	58.37288	60.2	61.96721	63.67742	65.33333	66.9375	68.49231	70	71.46269	72.88235	74.26087	75.6	76.90141
15	56.25	58.27869	60.24194	62.14286	63.98438	65.76923	67.5	69.1791	70.80882	72.3913	73.92857	75.42254	76.875	78.28767	79.66216
16	57.90476	60	62.03077	64	65.91045	67.76471	69.56522	71.31429	73.01408	74.66667	76.27397	77.83784	79.36	80.84211	82.28571
17	59.5	61.65672	63.75	65.78261	67.75714	69.67606	71.54167	73.35616	75.12162	76.84	78.51316	80.14286	81.73077	83.27848	84.7875
18	61.04348	63.25714	65.40845	67.5	69.53425	71.51351	73.44	75.31579	77.14286	78.92308	80.65823	82.35	84	85.60976	87.18072
19	62.54167	64.80822	67.01351	69.16	71.25	73.28571	75.26923	77.20253	79.0875	80.92593	82.71951	84.46988	86.17857	87.84706	89.47674
20	64	66.31579	68.57143	70.76923	72.91139	75	77.03704	79.02439	80.96386	82.85714	84.70588	86.51163	88.27586	90	91.68539
21	65.42308	67.78481	70.0875	72.33333	74.52439	76.66265	78.75	80.78824	82.77907	84.72414	86.625	88.48315	90.3	92.07692	93.81522
22	66.81481	69.21951	71.56627	73.85714	76.09412	78.27907	80.41379	82.5	84.53933	86.53333	88.48352	90.3913	92.25806	94.08511	95.87368
23	68.17857	70.62353	73.01163	75.34483	77.625	79.85393	82.03333	84.16484	86.25	88.29032	90.28723	92.24211	94.15625	96.03093	97.86735
24	69.51724	72	74.42697	76.8	79.12088	81.3913	83.6129	85.78723	87.91579	90	92.04124	94.04082	96	97.92	99.80198
25	70.83333	73.35165	75.81522	78.22581	80.58511	82.89474	85.15625	87.37113	89.54082	91.66667	93.75	95.79208	97.79412	99.75728	101.6827
26	72.12903	74.68085	77.17895	79.625	82.02062	84.36735	86.66667	88.92	91.12871	93.29412	95.41748	97.5	99.54286	101.5472	103.514



Five-speed manual transmission.

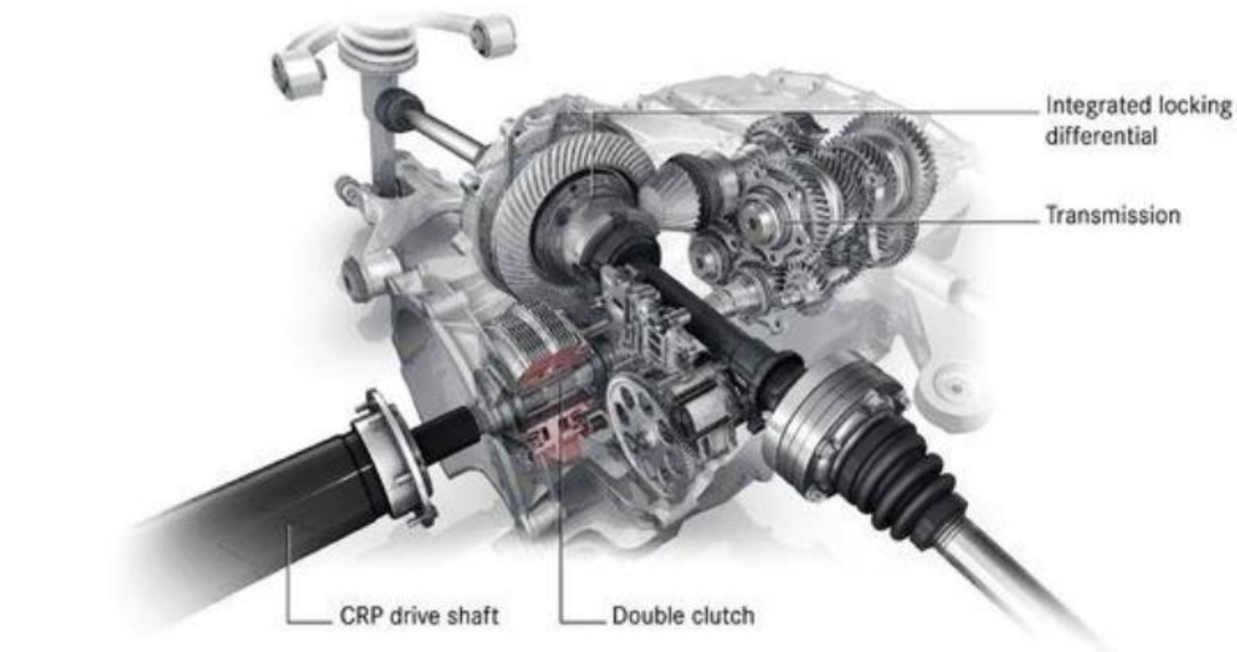
Gear	Ratio	RPM at Transmission Output Shaft with Engine at 3,000 rpm
1st	2.315:1	1,295
2nd	1.568:1	1,913
3rd	1.195:1	2,510
4th	1.000:1	3,000
5th	0.915:1	3,278

Select pd and tooth numbers. What is efficiency assuming Eo = 0.95?



Land Rover 5 speed transmission

Compact: AMG double-clutch transmission with integrated locking differential



Planetary Gear Trains:

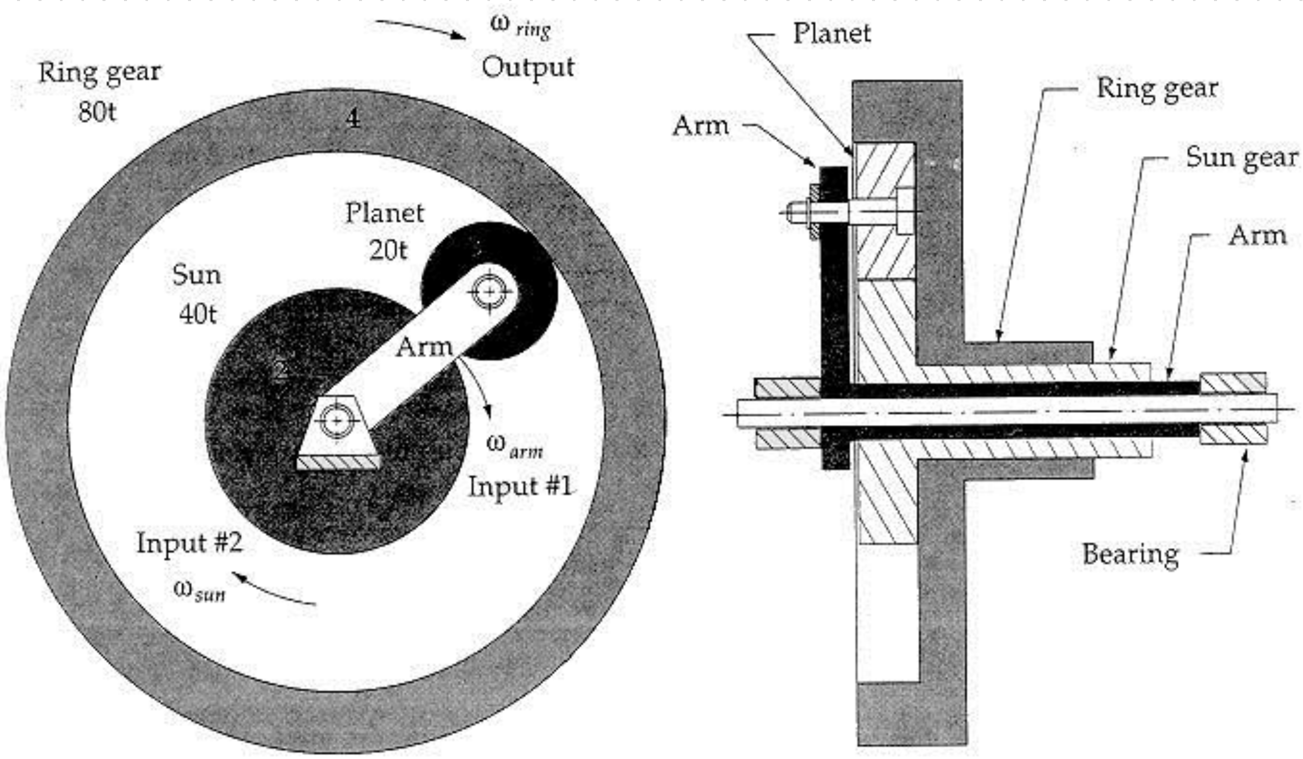
A Planetary gear train (see Fig. below) results when certain gears in the train (called the planet gears) have moving axes. The arm, while not a gear, is an essential part of the planetary because it defines the motion of the moving planet gear axes. The planetary is also unique to a standard gear train in that it requires two inputs to define one output (verify this using mobility). A good example is your car's differential, which has two inputs: one the drive-shaft, and the second a constraint between the two driven wheels provided by whatever you are driving on (e.g. dry pavement, one wheel on ice, etc.)

The planetary gear train consists of three parts:

1.

2.

3.



Planetary Gear Equation:

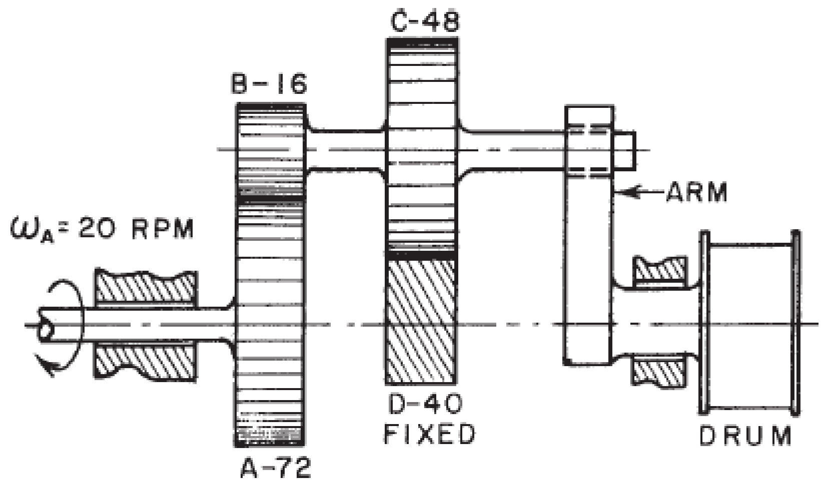
The planetary gear train equation must be used to solve the angular velocities of elements in the planetary. The equation is:

where:
f, and *l* identify two gears in the planetary (call them first and last),
a represents the arm,

ω_l, ω_f, ω_a,.

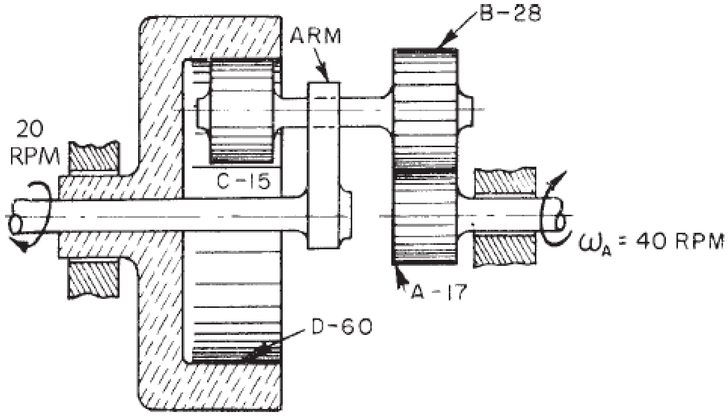
ω_{la}, ω_{fa}, $\omega_{la}/\ \omega_{fa}$,

Example: Planetary Gear Trains



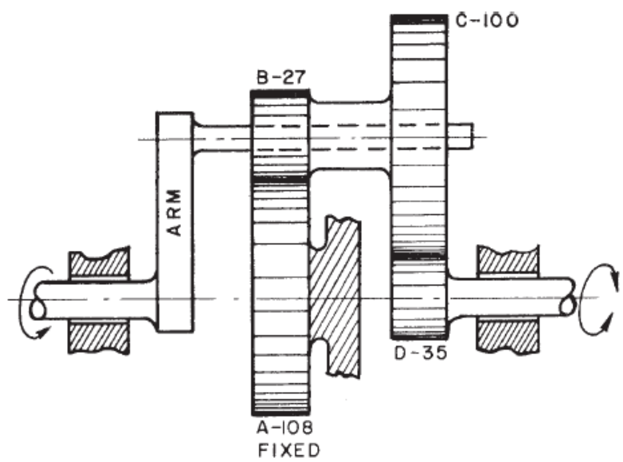
Given this gear train, find the speed and direction of the drum. What is train efficiency for Eo = 0.97 (Norton 9-35).

Example: Planetary Gear Train

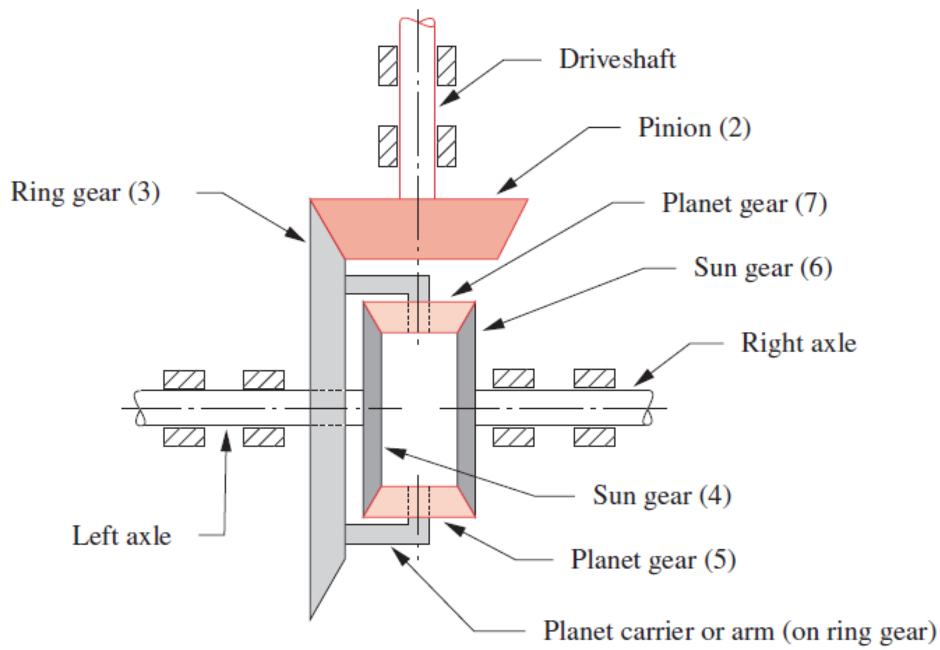


(b)

Given the planetary gear train above with inputs, what is the velocity of ring gear D?



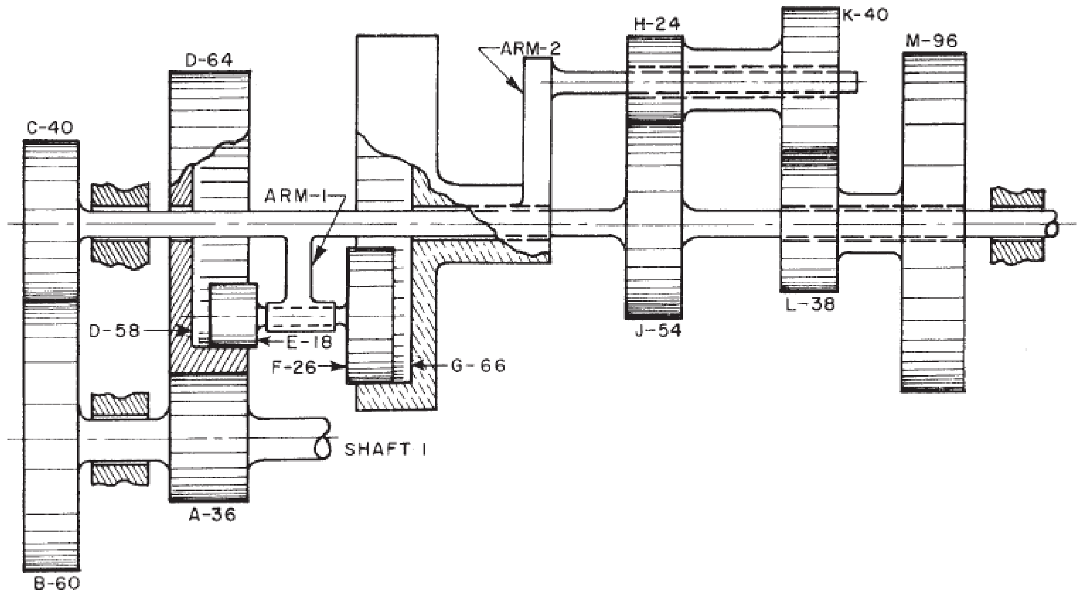
Given the planetary gear train above with inputs: Arm CCW at 50 rpm and gear A fixed to ground, find the speed of gear D. (Norton 9-37),



This is a schematic of an automotive differential. Notice that this is a planetary gear train. Assume that the engine is being driven at 2000 rpm, the transmission is in 4th gear (direct drive, 1:1 ratio) to the driveshaft.

Gears 4,5,6,7 = 15 teeth, Gear 3 = 43 teeth, Gear2 = 12 teeth, rear wheels = 40.64 cm

1) Assuming the vehicle is traveling straight down the road, what is its velocity
2) Vehicle is stuck in the mud, (right wheel in slippery mud, left wheel on firm pavement), what is speed of left and right wheel.
3) Turn the engine off, jack up the car, turn the left wheel + 1 rpm, what is right wheel speed?



Shaft 1 is input, find velocity of gear M.

Choosing *f, l,* and *a*:

Choosing elements for the first, last, and arm is the first step in solving a planetary. Solving will fall into one of the three following scenarios (remember that you must know two pieces of information to solve the planetary equation).

case i You want to find the arm velocity, (wa is not known) knowing the velocity of two gears: Choose f and l as the two known gears, and the arm as a, an unknown. Solve for *wa.*

case ii You want to find the velocity of a gear, and you know the velocity of the arm and one other gear:
Choose l as the desired unknown gear, choose f as the known gear and a as the known arm. Solve for *wl*.

case iii You want to find the velocity of a gear, and you know the velocity of two gears but not the arm.
First, choose f and l as the known gears and solve the arm velocity, *wa*. Then go to case ii.

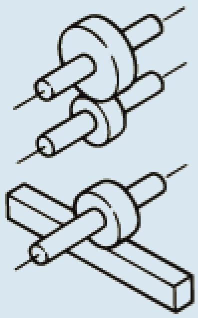
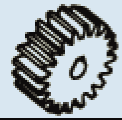
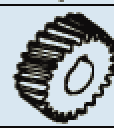
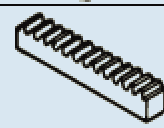
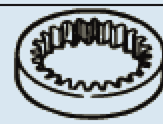
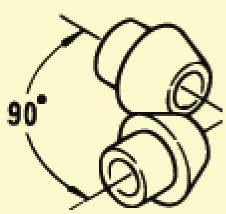
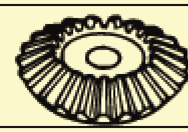
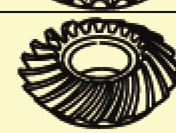
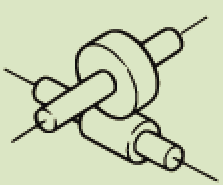
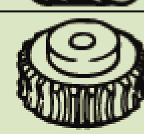
Mixed Gear trains:

A general gear train can include both fixed axis and planetary gear trains, or multiple planetaries. Solving systems like these requires using the procedures outlined above and looking for elements that share the same angular velocity between the mixed gear trains.

Gear Types:

A gear train consists of one or more gear sets intended to give a specific velocity ratio, or change direction of motion. Gear and gear train types can be grouped based on their application and tooth geometry.

Table I: Gear Types Grouped According to Shaft Arrangement

Categories of Gears	Types of Gears	Efficiency (%)	Isometrics
Parallel Axis Gears 	Spur Gear	98.0–99.5	
	Helical Gear		
	Rack・Helical Rack		
	Internal Gear		
Intersecting Axis Gears 	Miter Gear	98.0–99.0	
	Straight Bevel Gear		
	Spiral Bevel Gear		
Nonparallel and Nonintersecting Axis Gears 	Screw Gear (Crossed Helical Gear)	70.0–95.0	
	Worm	30.0–90.0	
	Worm Wheel		

(from khkgears.co.jp)

Spur gears (Fig. 1): Spur gears connect parallel shafts, have involute teeth that are parallel to the shafts, and can have either internal or external teeth. Notes:

1.
2. .
3.

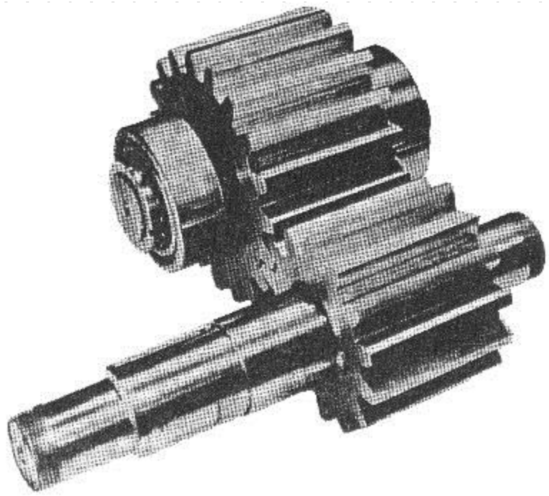


FIGURE 4.1 Spur gears. (Courtesy of Philadelphia Gear Works.)

Helical gears (Fig. 2): Helical gears also connect parallel shafts, but the involute teeth are cut at an angle (called the helix angle) to the axis of rotation. Note that two mating helical gears must have equal helix angle but opposite hand. These are found in automotive transmissions, and any application requiring high speed rotation and good performance. Notes:

1.
2.
3.
4.

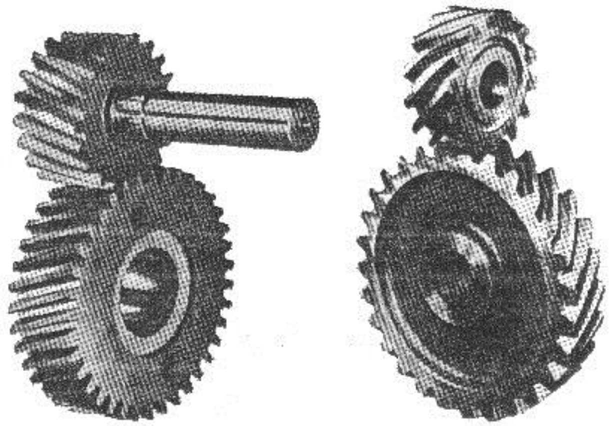

(a) (b)

FIGURE 6.16 Helical gears (a) for parallel shafts and (b) for crossed shafts. (Courtesy of D. O. James Gear Manufacturing Company.)

Herringbone gears (Fig. 3): To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces. These are called double helical or herringbone gears

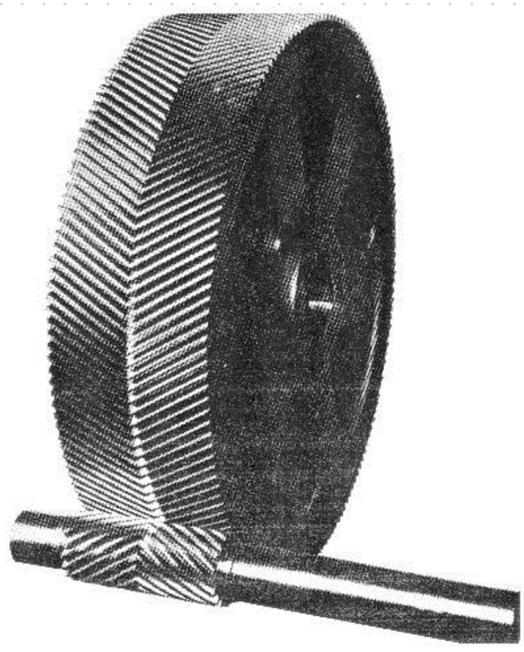


FIGURE 6.22 Herringbone gears. (Courtesy of D. O James Gear Manufacturing Company.)

Bevel gears (Fig. 4): Bevel gears connect intersecting axes, and come in several types (listed below). For bevel gears, the pitch surface is a cone, (it was a cylinder in spur and helical gears) and mating spiral gears can be modeled as two cones in rolling contact. Types of bevel gears:

1. Straight bevel: These are like spur gears, the teeth have no helix angle. Straight bevel gears can be
   a. Miter gears, equal size gears with a 90 degree shaft angle,
   b. Angular bevel gears, shaft angle other than 90 degrees, or
   c. Crown gears, one gear is flat, has a pitch angle of 90 degree.
2. Spiral bevel gears(Fig. 4a): Teeth have a spiral angle which gives performance improvements much like helical gears
3. Zerol bevel gears (Fig. 4b): Teeth are crowned, so that tooth contact takes place first at the tooth center.

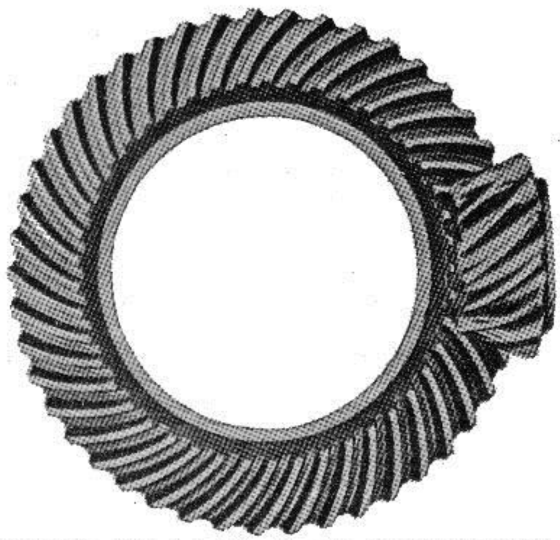
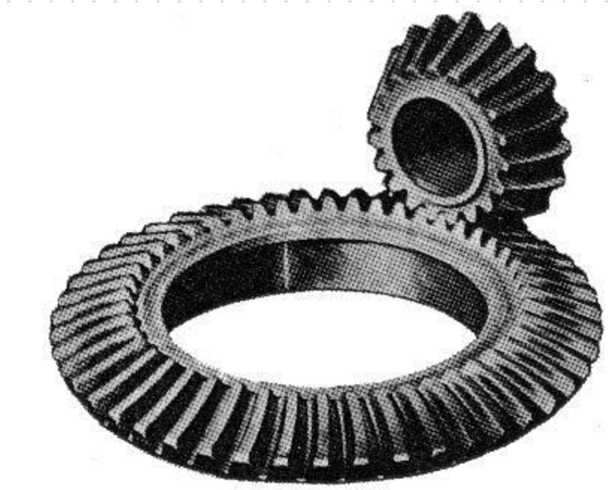
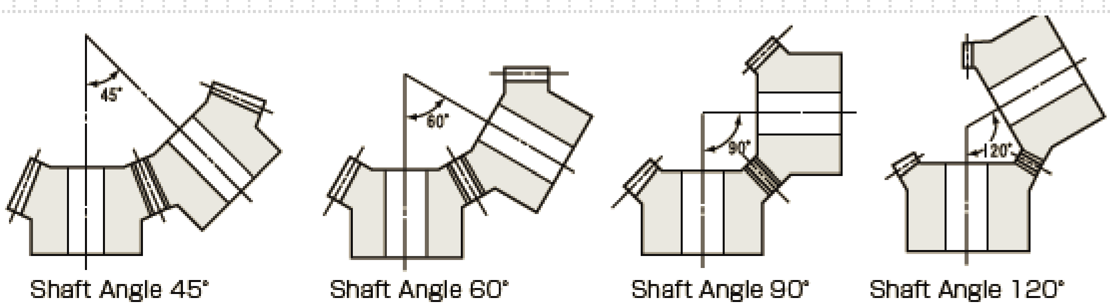


FIGURE 6.12 Spiral bevel gears. (Courtesy of Gleason Works.)



(*b*)

(*b*) Zerol bevel gears showing localized contact. (Courtesy

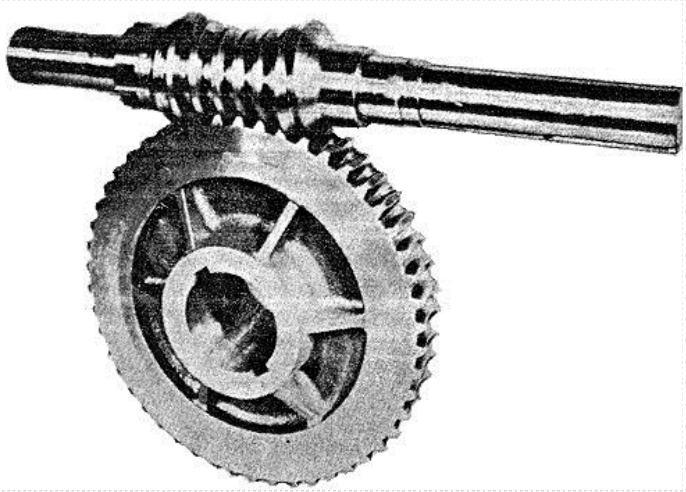


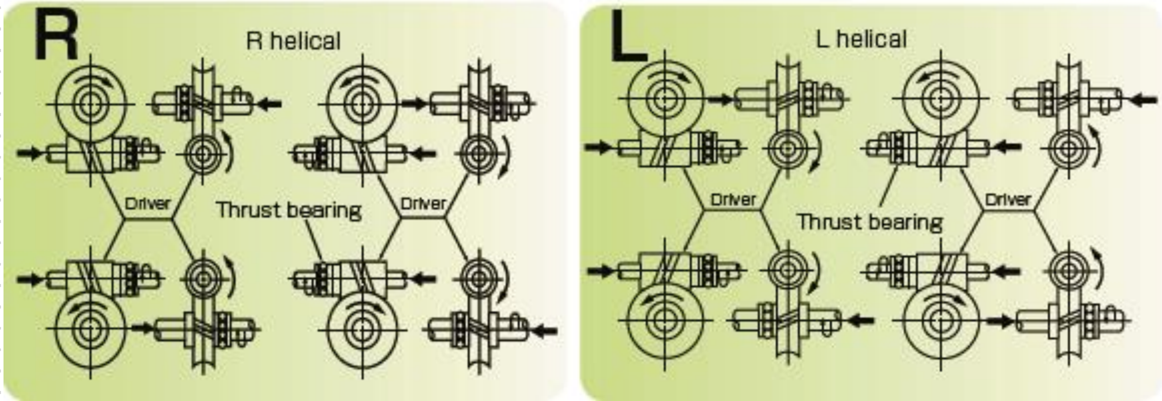
Hypoid gears (Fig. 5): Similar to spiral bevel gears, but connect non-parallel shafts that do not intersect. The pitch surface of a hypoid gear is a hyperboloid of revolution (rather than a cone, the pitch surface in bevel gears), hence the name.

FIGURE 6.14 Hypoid gears. (Courtesy of Gleason Works.)

Crossed helical gears (Fig. 6): Helical gears that connect skew shafts. The teeth have sliding motion and therefore lower efficiency. One application is connecting distributer to cam shaft in pre-electronic ignition vehicles.

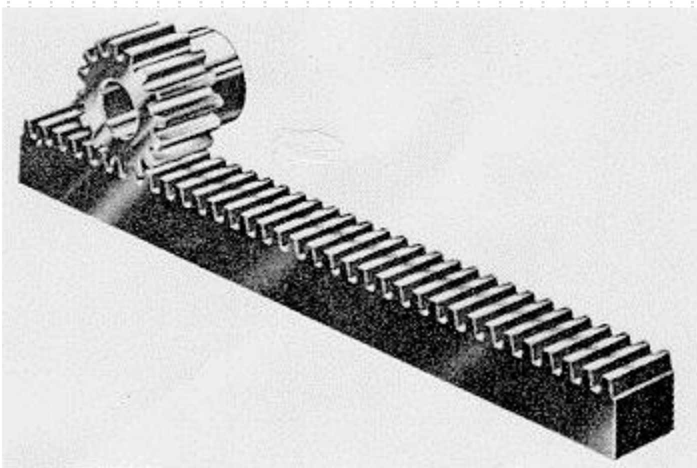
Worm Gears (Fig. 7): The driving gear is called a worm, and typically has 1, 2, or four teeth. The low number of teeth on the worm can result in a very large velocity ratio. These can also be designed to be non-backdriveable, and can carry high loads. Because of sliding action, efficiency is low.



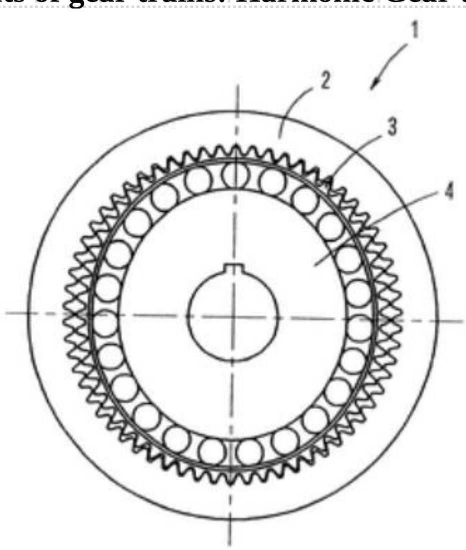


(from khkgears.co.jp)

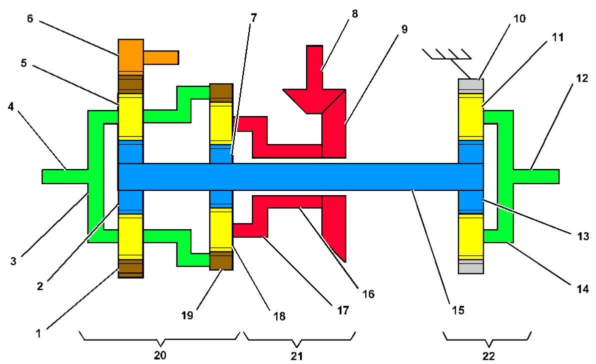
Rack and Pinion (Fig. 8): These transmit rotary motion (from the pinion) to translational motion (of the rack). The rack is a gear with infinite radius; its teeth, although flat sided, are involute. The rack and pinion is commonly used in steering units and jacks.



Variants of gear trains: Harmonic Gear trains



Typical Components for a Differential Steering System
(1) Ring Gear
(2) Sun Gear
(3) Carrier
(4) Outer Axle Shaft
(5) Planetary Gears
(6) Steering Motor
(7) Sun Gear
(8) Transmission Pinion
(9) Bevel Gear
(10) Stationary Ring Gear
(11) Planetary Gears
(12) Outer Axle Shaft
(13) Sun Gear
(14) Carrier
(15) Inner Axle Shaft
(16) Bevel Gear Shaft
(17) Carrier
(18) Planetary Gears(19) Ring Gear



Differential Steering System
(1) Ring Gear
(2) Sun Gear
(3) Carrier
(4) Outer Axle Shaft
(5) Planetary Gears
(6) Input from the Steering Motor
(7) Sun Gear
(8) Transmission Pinion
(9) Bevel Gear
(10) Stationary Ring Gear
(11) Planetary Gears
(12) Outer Axle Shaft
(13) Sun Gear
(14) Carrier
(15) Inner Axle Shaft
(16) Bevel Gear Shaft
(17) Carrier
(18) Planetary Gears
(19) Ring Gear
(20) Steering Differential
(21) Bevel Gear Set(22) Planetary Gear Train

Steering differential (20) receives power from the following two components: