Expansion and isothermal coefficients

For a perfect gas;

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

$$= \frac{1}{V}\left(\frac{\partial (nRT/p)}{\partial T}\right)_p$$

$$= \frac{1}{V} \times \frac{nR}{p}$$

$$= \frac{nR}{nRT} = \frac{1}{T}$$

	$\alpha/(10^{-4}\ \mathrm{K}^{-1})$	$\kappa_T/(10^{-6}\ \mathrm{bar}^{-1})$
Liquids:		
Benzene	12.4	90.9
Water	2.1	49.0
Solids:		
Diamond	0.030	0.185
Lead	0.861	2.18

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

expansion coefficient

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

isothermal compressibility

For a perfect gas;

$$\kappa_T = 1/p$$

Cp - Cv

$$dU = \pi_T dV + C_V dT$$

Cp - Cv

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for a perfect gas:

$$\left(\frac{dU}{dT}\right)_V = \left(\frac{dU}{dT}\right)_P = C_v$$

Since the internal energy U is a function of temperature only

Cp - Cv

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Definition of C_p

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$$C_p - C_V = \frac{\alpha^2 TV}{\kappa_T}$$

for any other gas

Changes in enthalpy

$$dH = \left(\frac{\partial H}{\partial p}\right)_T dp + \left(\frac{\partial H}{\partial T}\right)_p dT$$

The variation of enthalpy with temperature and pressure

C_V

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

internal pressure

Changes in enthalpy

$$dH = \left(\frac{\partial H}{\partial p}\right)_T dp + \left(\frac{\partial H}{\partial T}\right)_p dT$$

The variation of enthalpy with temperature and pressure

$$dH = -\mu C_p dp + C_p dT$$

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Joule–Thomson coefficient

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The variation of enthalpy with temperature and pressure

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Joule–Thomson coefficient

$$\mu = \left(\frac{\partial T}{\partial p}\right)_H$$

the variation of temperature with pressure at constant enthalpy

Joule-Thomson effect

The Joule-Thomson effect (or Joule-Kelvin effect) describes a real gas's temperature change when it undergoes an isenthalpic process.

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LAW: Show that the Joule-Thomson (JT) effect is an isenthalpic process using gas flowing through a porous plug or a throttle.

Joule-Thomson coefficient

$$\mu = \left(\frac{\partial T}{\partial p} \right)_H$$

$$\mathrm{d}H = -\mu C_p \mathrm{d}p + C_p \mathrm{d}T$$

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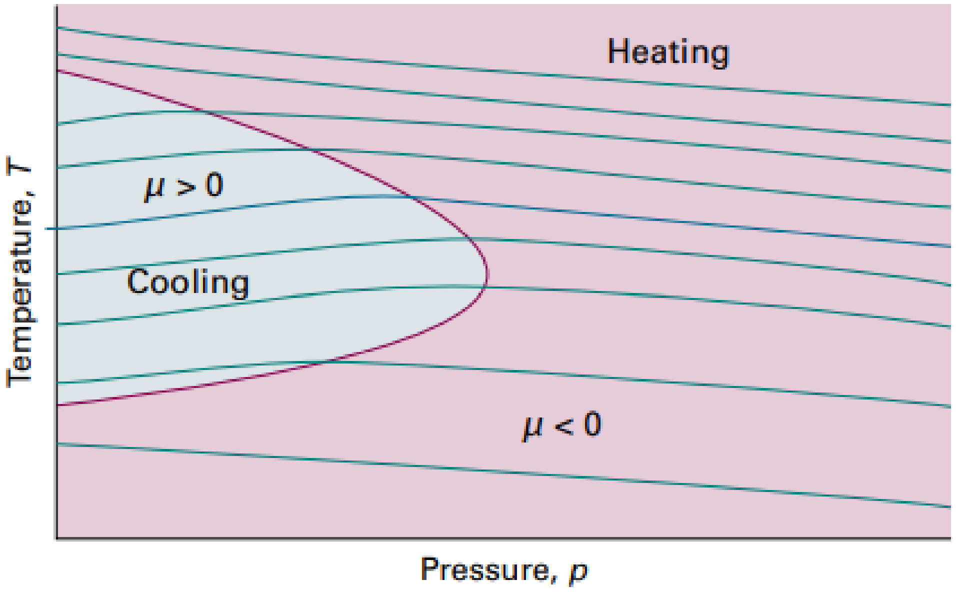

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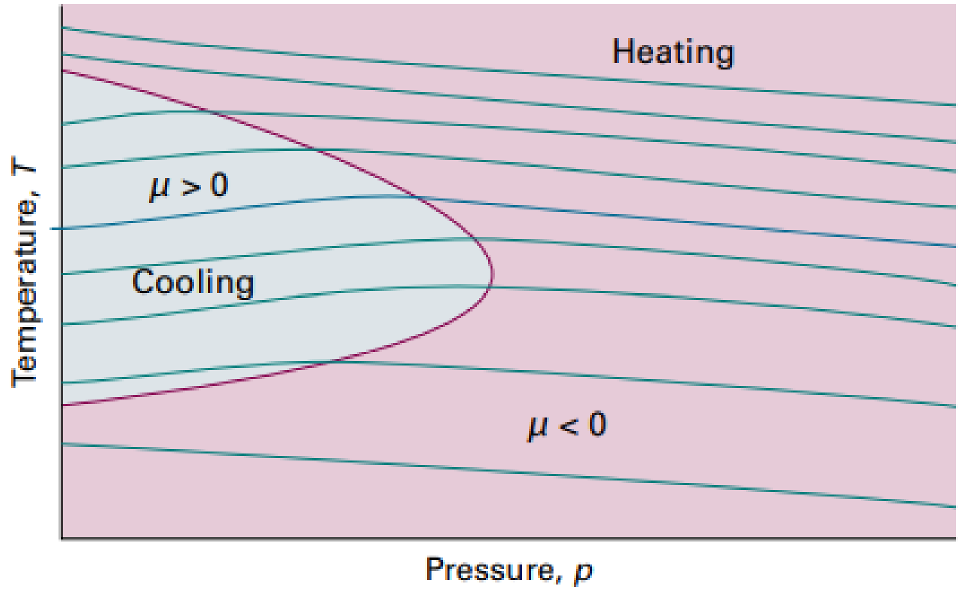

The temperature corresponding to the boundary at a given pressure is the 'inversion temperature' of the gas at that pressure

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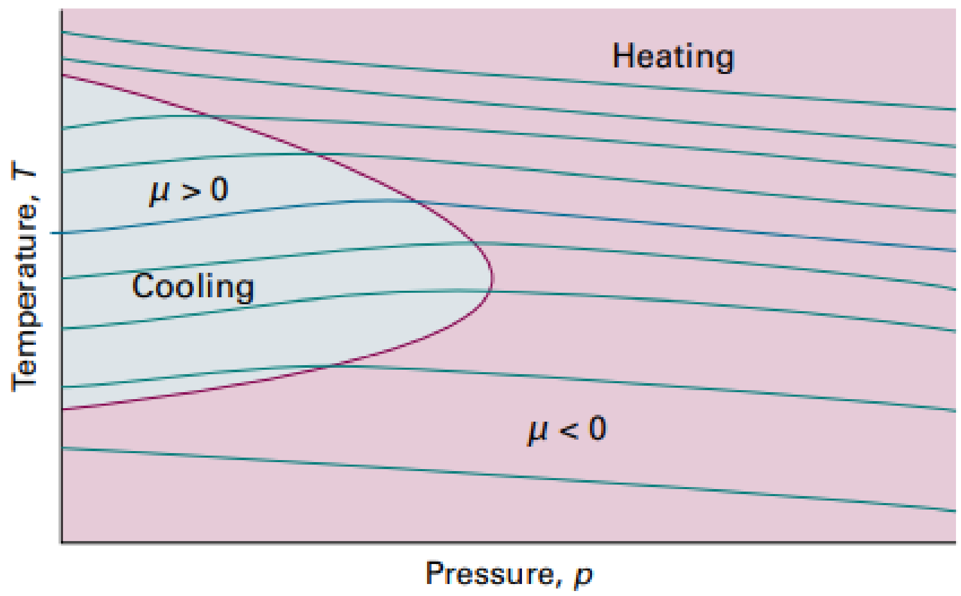

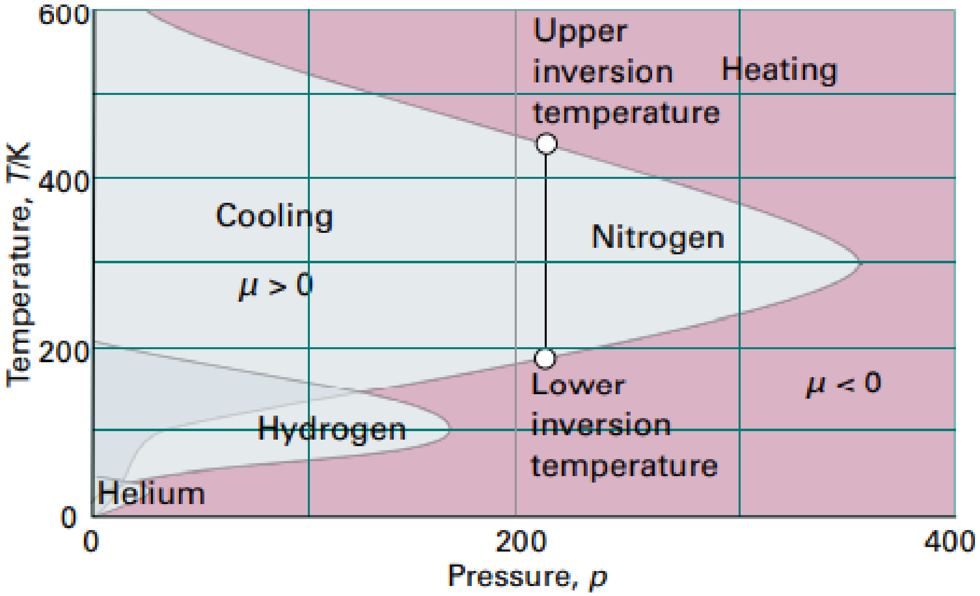

The inversion temperature curve runs through the points of the isenthalps where their slope changes from negative to positive.

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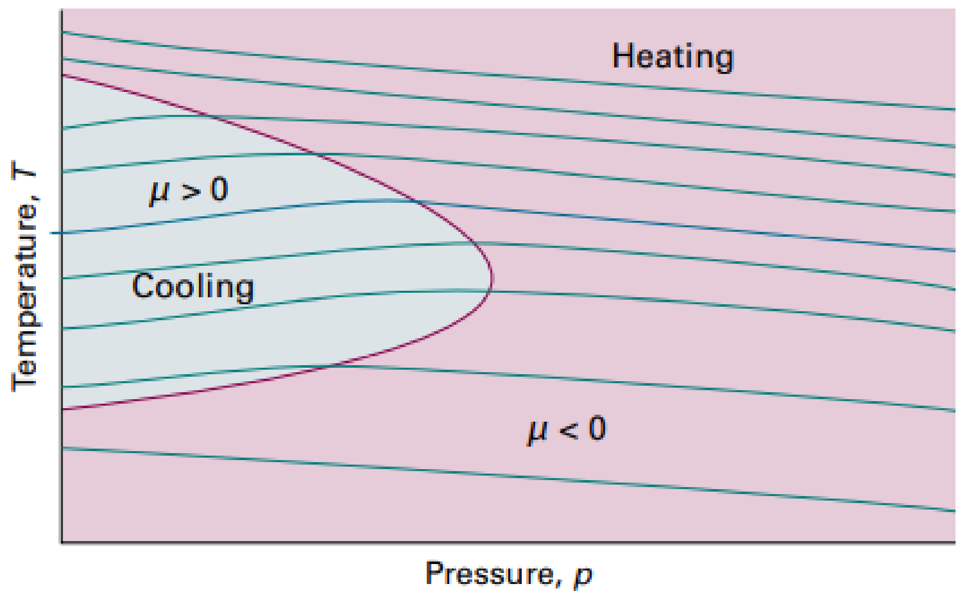

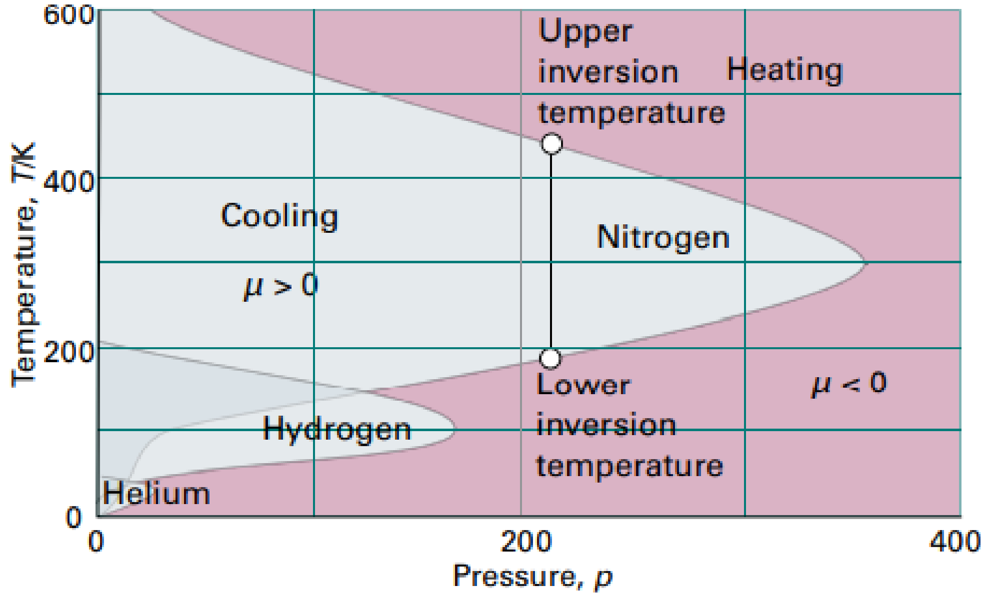

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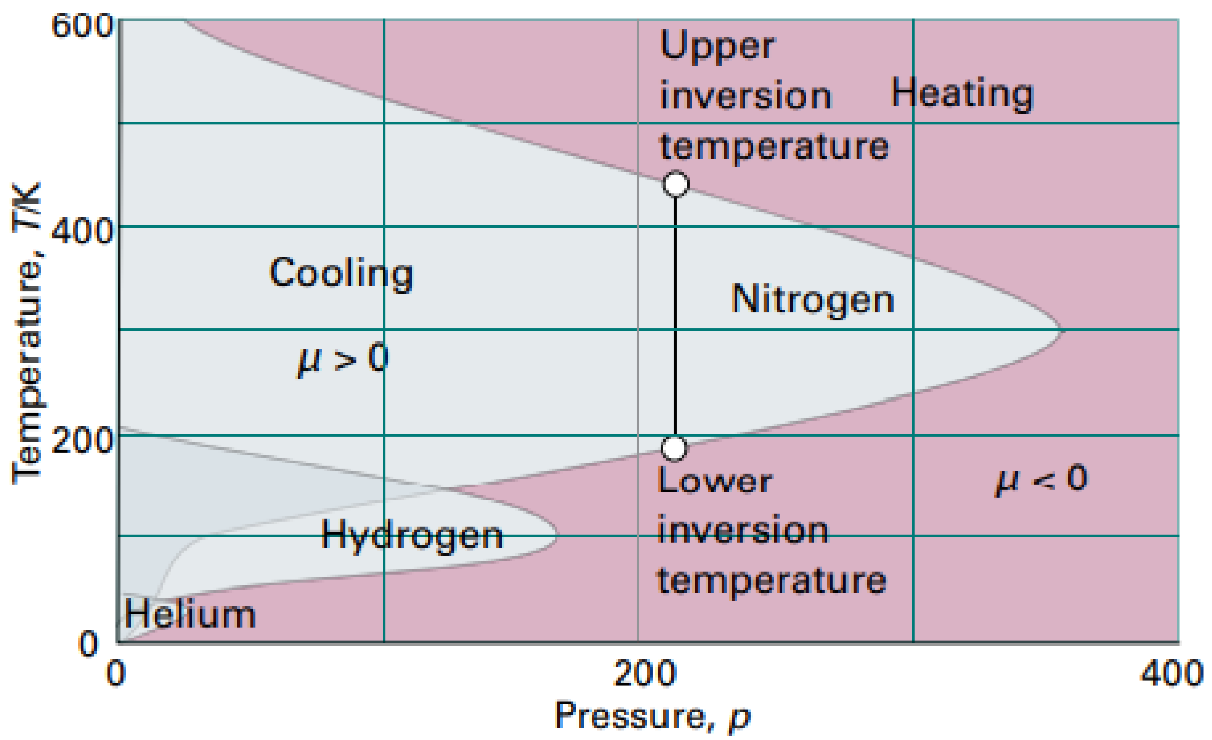
Nitrogen:
Has a high inversion temperature, allowing cooling over a wide temperature range.
•Is easily liquefied using the Joule-Thomson effect.

Hydrogen and Helium:
•Both have very low inversion temperatures (below room temperature).
•At room temperature, expanding hydrogen or helium causes heating!
•To cool them via the JT effect, they must first be precooled below their inversion temperatures using other methods.



Focus 2: The First Law

- Internal Energy
- Enthalpy
- Thermochemistry
- State functions
- Adiabatic changes