

## Focus 3: The Second and Third Laws

Entropy

Entropy changes in processes

Entropy measurement

Free energy

Combining 1<sup>st</sup> and 2<sup>nd</sup> laws

# The fundamental equation

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## The fundamental equation

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$$dU = TdS - pdV$$

This equation applies to any change in  $U$ —reversible or irreversible—of a closed system that does no additional (non-expansion) work!

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$$\left( \frac{\partial U}{\partial S} \right)_V = T \quad \left( \frac{\partial U}{\partial V} \right)_S = -p$$

# Maxwell relations

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An infinitesimal change in a function  $f(x,y)$

$$df = gdx + hdy$$

where:

- $g$  is the partial derivative of  $f$  with respect to  $x$ , i.e.,  $g = \frac{\partial f}{\partial x}$ .
- $h$  is the partial derivative of  $f$  with respect to  $y$ , i.e.,  $h = \frac{\partial f}{\partial y}$ .

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For  $df$  to be an **exact differential**, meaning it represents the total derivative of some function  $f(x, y)$ , the mixed second partial derivatives must be equal:

$$\left(\frac{\partial g}{\partial y}\right)_x = \left(\frac{\partial h}{\partial x}\right)_y$$

- This is a necessary and sufficient condition for the differential  $df$  to be exact, which is fundamental in vector calculus, thermodynamics, and differential equations.

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$$dU = TdS - pdV$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$







Using Maxwell relations















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# Variations of Gibbs energy with temperature

$$\left( \frac{\partial G/T}{\partial T} \right)_p = - \frac{H}{T^2}$$

Gibbs–Helmholtz equation

- The equation relates Gibbs free energy to enthalpy ( $H$ ) and temperature.
- It shows how  $G/T$  varies with temperature, which is important in determining equilibrium constants and spontaneity of reactions.

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If the change in enthalpy is known,  
then how the change in Gibbs energy varies  
with temperature is also known.

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condensed phase

$$G_m(p_f) = G_m(p_i) + V_m \int_{p_i}^{p_f} dp$$









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if  $p_i = p^\ominus$  (the standard pressure of 1 bar)

set  $p_f = p$

