# Focus 3: The Second and Third Laws

Entropy

Entropy changes in processes

Entropy measurement

Gibbs free energy

Combining 1<sup>st</sup> and 2<sup>nd</sup> law





Heat solid to its melting point  $S_{\rm m}(T) = S_{\rm m}(0) + \int_{0}^{T_{\rm f}} \frac{C_{p,{\rm m}}({\rm s},T')}{T'} {\rm d}T'$ 

υ





υ

















Heat solid  
to its  
melting point  
$$S_{\rm m}(T) = S_{\rm m}(0) + \int_{0}^{T_{\rm f}} \frac{C_{p,{\rm m}}({\rm s},T')}{T'} {\rm d}T' + \frac{\Delta_{\rm fus}H}{T_{\rm f}}$$





The standard molar entropy of nitrogen gas at 25 °C

	<b>Contribution to</b> $S_{\rm m}^{\oplus}/(J{\rm K}^{-1}{\rm mol}^{-1})$		
Integration, from 10 K to 35.61 K	25.25		
Phase transition at 35.61 K	6.43		
Integration, from 35.61 K to 63.14 K	23.38		F
Fusion at 63.14 K	11.42		
Integration, from 63.14 K to 77.32 K	11.41	S	$\Delta_{fus}H/T_{f}$
Vaporization at 77.32 K	72.13		
Integration, from 77.32 K to 298.15 K	39.20	Solid	Liquid
		7	T, T

#### The standard molar entropy of nitrogen gas at 25 °C



### The standard molar entropy of nitrogen gas at $25\,^{\circ}\text{C}$

	Contribution to $S_{\rm m}^{\oplus}/($	$(\mathbf{J} \mathbf{K}^{-1} \mathbf{mol}^{-1})$
Debye extrapolation	1.92	
Integration, from 10K to 35.61K	25.25	
Phase transition at 35.61 K	6.43	
Integration, from 35.61 K to 63.14 K	23.38	
Fusion at 63.14 K	11.42	
Integration, from 63.14 K to 77.32 K	11.41	
Vaporization at 77.32 K	72.13	
Integration, from 77.32 K to 298.15 K	39.20	

 $\blacktriangleright$  Debye  $T^3$  law

Assume that the heat capacity of a nonmetallic solid is proportional to  $T^3$ 

## The standard molar entropy of nitrogen gas at $25\,^\circ\text{C}$

	Contribution to $S_{\rm m}^{\oplus}/(J  {\rm K}^{-1}  {\rm mol}^{-1})$	
Debye extrapolation	1.92	$\longrightarrow$ Debye $T^3$ law
Integration, from 10 K to 35.61 K	25.25	Assume that the heat canacity of a non-
Phase transition at 35.61 K	6.43	metallic solid is proportional to $T^3$
Integration, from 35.61 K to 63.14 K	23.38	
Fusion at 63.14 K	11.42	
Integration, from 63.14 K to 77.32 K	11.41	
Vaporization at 77.32 K	72.13	
Integration, from 77.32 K to 298.15 K	39.20	
Correction for gas imperfection	0.92	
Total	192.06	

Therefore,  $S_{\rm m}^{\oplus}(298.15\,{\rm K}) = S_{\rm m}(0) + 192.1\,{\rm J\,K}^{-1}\,{\rm mol}^{-1}$ 

$$S(T_2) = S(T_1) + \int_{T_1}^{T_2} \frac{C_p(T)}{T} dT$$

$$C_{p,m}(T) = aT^{3}$$

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; T = 4.2

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$$= S_{\rm m}(0) + \frac{1}{3}aT^3 = S_{\rm m}(0) + \frac{1}{3}C_{p,\rm m}(T) \qquad ; T = 4.2$$

The molar constant-pressure heat capacity of a certain nonmetallic solid at 4.2 K is  $0.43 \text{ J} \text{ K}^{-1} \text{ mol}^{-1}$ . What is its molar entropy at that temperature?

$$C_{p,m}(T) = aT^{3}$$

$$S(T_{2}) = S(T_{1}) + \int_{T_{1}}^{T_{2}} \frac{C_{p}(T)}{T} dT$$

$$S_{\rm m}(T) = S_{\rm m}(0) + \int_0^T \frac{aT'^3}{T'} dT'$$
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$$= S_{\rm m}(0) + \frac{1}{3}aT^3 = S_{\rm m}(0) + \frac{1}{3}C_{p,\rm m}(T)$$

 $S_{\rm m}(4.2\,{\rm K}) = S_{\rm m}(0) + 0.14\,{\rm J}\,{\rm K}^{-1}\,{\rm mol}^{-1}$ 

The entropy of a pure substance in a perfect crystalline state at zero temperature is zero

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$$T = 0, \ \mathcal{W} = 1$$

There is only one way of arranging the molecules when they are all in the ground state

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$$T = 0, \mathcal{W} = 1$$
 S = k ln W

There is only one way of arranging the molecules when they are all in the ground state



Structures of Cage, Prism, and Book Isomers of Water Hexamer from Broadband Rotational Spectroscopy Cristóbal Pérez et al. Science 336, 897 (2012); DOI: 10.1126/science.1220574





REPORTS

CAGE PRISM BOOK -87.4 -100.6 -88.8 -93.8 kJ/mol -61.7 -68.5 2.83(1)2.73(1)2.84 2.72 2.79(1) 2.82(1) -56.4 2.82 2.78 -80.5 2.70(1) 2.79(1) 2.96(1) 2.81(1) 2.98(1) 2.73(1) -60.5 -64.5 2.75(1) 2.76 -60.1 2.79(1) -82.9 2.79 2.82(1) 2.72(1) -90.5 -68.7 -75.8 -71.5 -60.4 2.822.72 -93.3 top ring view from the right apex view towards the bottom edge 2.93(1) 2.81(1) 2.92 2.81 2.71(1) 2.71-88.8 -93.8 \*\*\*\*\* 2.99(1) 3.01(1) 2.96(1) bottom ring 3.02 3.02 2.96-75.8 O...O distance: -68.7 2.98(1)2.83(1) -3.002.83 -2.98(1) exp., r0 2.97 calc., vibrationally averaged 2.99(1) 2.99

Fig. 4. The experimental ro-analysis structures are shown for the three water hexamer isomers. The dashed lines indicate the hydrogen-bonding network. The experimental 0...0 bond distances. in Angstroms, are given in bold with the theoretical values from the vibrationally averaged structures below. The detachment energy for each water molecule in the duster is given in red.

Chemical Physics Letters 571 (2013) 1–15 Contents lists available at SciVerse ScienceDirect



Chemical Physics Letters



Frontiers Article

Broadband Fourier transform rotational spectroscopy for structure determination: The water heptamer

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Figure 3. Several low-energy oxygen-atom frameworks of the water heptamer (energy differences in kcal/mol).



#### Water Clusters

DOI: 10.1002/anie.201407447

# Hydrogen Bond Cooperativity and the Three-Dimensional Structures of Water Nonamers and Decamers\*\*

Cristóbal Pérez, Daniel P. Zaleski, Nathan A. Seifert, Berhane Temelso, George C. Shields,\* Zbigniew Kisiel,\* and Brooks H. Pate\*



# Standard Third Law entropies

## Standard Third Law entropies

When the substance is in its standard state at the temperature, T

$$S(T_{f}) = S(T_{i}) + C_{p} \int_{T_{i}}^{T_{f}} \frac{dT}{T}$$

$$0 \text{ J } \text{K}^{-1} \text{ mol}^{-1}$$

$$(T_{i} = 0 \text{ K})$$

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$$S(T_{f}) = S(T_{i}) + C_{p} \int_{T_{i}}^{T_{f}} \frac{dT}{T}$$
  
0 J K<sup>-1</sup> mol<sup>-1</sup>  
(T\_{i} = 0 K)

#### $S_{\rm m}^{\Theta}/({\rm J}\,{\rm K}^{-1}\,{\rm mol}^{-1})$ Solids Graphite, C(s) 5.7 Diamond, C(s) 2.4 Sucrose, $C_{12}H_{22}O_{11}(s)$ 360.2 Liquids Benzene, $C_6H_6(l)$ 173.3 Water, H<sub>2</sub>O(l) 69.9 Gases Methane, $CH_4(g)$ 186.3 Carbon dioxide, CO<sub>2</sub>(g) 213.7 Hydrogen, $H_2(g)$ 130.7 Helium, He(g) 126.2 Ammonia, NH<sub>3</sub>(g) 192.4

#### Standard Third-Law entropies at 298 K



All substances being in their standard states at the specified temperature



All substances being in their standard states at the specified temperature





All substances being in their standard states at the specified temperature

$$\Delta_{\rm r} S^{\oplus} = \sum_{\rm J} v_{\rm J} S^{\oplus}_{\rm m}({\rm J})$$

Question: Calculate the standard molar reaction entropy of the formation of water



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 $H_2(g) + 1/2 O_2(g) \rightarrow H_2O(I)$ 



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Question: Calculate the standard molar reaction entropy of the formation of water

 $\Delta_{\mathbf{r}}S^{\ominus} = S^{\ominus}_{\mathbf{m}}(\mathbf{H}_{2}\mathbf{O},\mathbf{l}) - \{S^{\ominus}_{\mathbf{m}}(\mathbf{H}_{2},\mathbf{g}) + \frac{1}{2}S^{\ominus}_{\mathbf{m}}(\mathbf{O}_{2},\mathbf{g})\}$ 

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$$\Delta_{\rm r} S^{\oplus} = \sum_{\rm J} v_{\rm J} S^{\oplus}_{\rm m}({\rm J})$$

Question: Calculate the standard molar reaction entropy of the formation of water

$$\begin{split} \Delta_{\mathbf{r}} S^{\ominus} &= S^{\ominus}_{\mathbf{m}}(\mathbf{H}_{2}\mathbf{O},\mathbf{l}) - \{S^{\ominus}_{\mathbf{m}}(\mathbf{H}_{2},\mathbf{g}) + \frac{1}{2}S^{\ominus}_{\mathbf{m}}(\mathbf{O}_{2},\mathbf{g})\} \\ &= 69.9 \, \mathrm{J} \, \mathrm{K}^{-1} \, \mathrm{mol}^{-1} - \{130.7 + \frac{1}{2}(205.1)\} \, \mathrm{J} \, \mathrm{K}^{-1} \, \mathrm{mol}^{-1} \\ &= -163.4 \, \mathrm{J} \, \mathrm{K}^{-1} \, \mathrm{mol}^{-1} \end{split}$$

$$S^{\ominus}(\mathrm{H}^{+},\mathrm{aq})=0$$

lons in solution [convention]

at all temperatures!

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 $S_{\rm m}^{\scriptscriptstyle \ominus}/({\rm J~K}^{\!-1}\,{
m mol}^1)^{\dagger}$ 

Cu<sup>+</sup>(aq) +40.6 Cu<sup>2+</sup>(aq) -99.6

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 $S_{\mathrm{m}}^{\ominus}/(\mathrm{J} \mathrm{K}^{-1} \mathrm{mol}^{1})^{\dagger}$ 

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=> disorder of the solution is decreased

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at all temperatures!

$S_{\rm m}^{\scriptscriptstyle \ominus}$	(J	<b>K</b> <sup>-1</sup>	$\mathbf{mol}^{1})^{\dagger}$

10 1

Cu <sup>+</sup> (aq)	+40.6
Cu <sup>2+</sup> (aq)	-99.6
Mg <sup>2+</sup> (aq)	-128
Cl <sup>-</sup> (aq)	57

=> disorder of the solution is decreased

$$S_{\rm m}(T_2) = S_{\rm m}(T_1) + \int_{T_1}^{T_2} \frac{C_{p,{\rm m}}(T)}{T} {\rm d}T$$

$$\Delta_{\mathbf{r}} S^{\ominus}(T_2) = \Delta_{\mathbf{r}} S^{\ominus}(T_1) + \int_{T_1}^{T_2} \frac{\Delta_{\mathbf{r}} C_p^{\ominus}}{T} dT$$

$$\Delta_{\mathbf{r}} C_{p}^{\ominus} = \sum_{\mathbf{r}} \mathbf{v}_{\mathbf{J}} C_{p,\mathbf{m}}^{\ominus}(\mathbf{J})$$

$$\mathbf{\Delta}_{\mathbf{r}} S^{\ominus}(T_{2}) = \Delta_{\mathbf{r}} S^{\ominus}(T_{1}) + \int_{T_{1}}^{T_{2}} \frac{\Delta_{\mathbf{r}} C_{p}^{\ominus}}{T} dT$$

$$S_{\rm m}(T_2) = S_{\rm m}(T_1) + \int_{T_1}^{T_2} \frac{C_{p,{\rm m}}(T)}{T} dT$$

 $\Delta_{\mathbf{r}} C_p^{\oplus}$  is the difference of the molar heat capacities of products and reactants under standard conditions weighted by the stoichiometric numbers that appear in the chemical equation:

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 $\Delta_{\mathbf{r}} C_p^{\ominus} = \sum_{\mathbf{J}} v_{\mathbf{J}} C_{p,\mathbf{m}}^{\ominus} (\mathbf{J})$ 

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If  $\Delta_r C_p \ominus$  is independent of temperature in the range  $T_1$  to  $T_2$ :

$$\Delta_{\mathbf{r}} S^{\ominus}(T_2) = \Delta_{\mathbf{r}} S^{\ominus}(T_1) + \Delta_{\mathbf{r}} C_p^{\ominus} \ln \frac{T_2}{T_1}$$

$$\Delta_{\mathbf{r}} S^{\ominus}(T_2) = \Delta_{\mathbf{r}} S^{\ominus}(T_1) + \int_{T_1}^{T_2} \frac{\Delta_{\mathbf{r}} C_p^{\ominus}}{T} dT$$

$$\Delta_{\rm r} C_p^{\ominus} = \sum_{\rm I} v_{\rm J} C_{p,{\rm m}}^{\ominus}({\rm J})$$

$$\int_{T_1}^{T_2} \frac{\Delta_{\rm r} C_p^{\ominus}}{T} dT$$

## Calculation

$$\Delta_{\mathbf{r}} S^{\ominus}(T_2) = \Delta_{\mathbf{r}} S^{\ominus}(T_1) + \int_{T_1}^{T_2} \frac{\Delta_{\mathbf{r}} C_p^{\ominus}}{T} dT$$

The standard reaction entropy for  $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g)$ at 298 K is  $-44.42 J K^{-1} mol^{-1}$ , and the molar heat capacities at constant pressure of the molecules are  $H_2O(g)$ : 33.58 J K<sup>-1</sup> mol<sup>-1</sup>;  $H_2(g)$ : 28.84 J K<sup>-1</sup> mol<sup>-1</sup>;  $O_2(g)$ : 29.37 J K<sup>-1</sup> mol<sup>-1</sup>.

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$$\Delta_{\rm r} C_{\rm p}^{\Theta} = C_{\rm p,m}^{\Theta} ({\rm H}_{2}{\rm O},{\rm g}) - C_{\rm p,m}^{\Theta} ({\rm H}_{2},{\rm g}) - \frac{1}{2} C_{\rm p,m}^{\Theta} ({\rm O}_{2},{\rm g})$$
$$= -9.94 \,{\rm J} \,{\rm K}^{-1} \,{\rm mol}^{-1}$$

## Calculation

 $\Delta_{\mathbf{r}} S^{\ominus}(T_2) = \Delta_{\mathbf{r}} S^{\ominus}(T_1) + \Delta_{\mathbf{r}} C_p^{\ominus} \ln \frac{T_2}{T_1}$ 

The standard reaction entropy for  $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g)$ at 298 K is  $-44.42 \text{ J K}^{-1} \text{ mol}^{-1}$ , and the molar heat capacities at constant pressure of the molecules are  $H_2O(g)$ :  $33.58 \text{ J K}^{-1} \text{ mol}^{-1}$ ;  $H_2(g)$ : 28.84 J K<sup>-1</sup> mol<sup>-1</sup>;  $O_2(g)$ : 29.37 J K<sup>-1</sup> mol<sup>-1</sup>.

$$\Delta_{\rm r} C_{p}^{\ominus} = C_{p,{\rm m}}^{\ominus} ({\rm H}_{2}{\rm O},{\rm g}) - C_{p,{\rm m}}^{\ominus} ({\rm H}_{2},{\rm g}) - \frac{1}{2} C_{p,{\rm m}}^{\ominus} ({\rm O}_{2},{\rm g})$$
$$= -9.94 \,{\rm J} \,{\rm K}^{-1} \,{\rm mol}^{-1}$$

 $\Delta_{\rm r} S^{\oplus}(373\,{\rm K}) = -44.42\,{\rm J}\,{\rm K}^{-1}\,{\rm mol}^{-1} + (-9.94\,{\rm J}\,{\rm K}^{-1}\,{\rm mol}^{-1}) \times \ln\frac{373\,{\rm K}}{298\,{\rm K}}$  $= -46.65\,{\rm J}\,{\rm K}^{-1}\,{\rm mol}^{-1}$ 

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- Entropy measurement
- Free energy
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