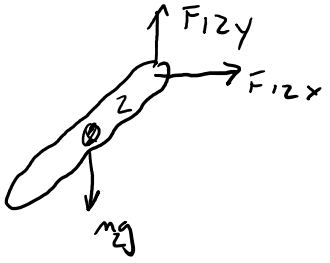
Chap 2 Practice:

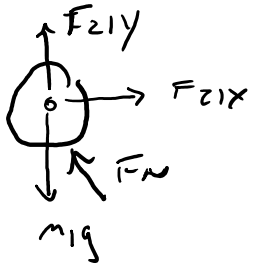
Monday, June 12, 2017 2:55 PM

2-1 Create a FBD of the pendulum link:

Solution:

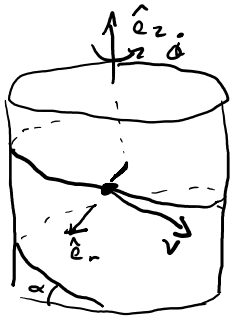
2 bodies: wheel & link





w/ F_N the normal force

2-6



find Jacobian:

$$\vec{V} = \frac{d}{dt}(\vec{r}) = \frac{d}{dt}(R\hat{e}_r + z\hat{e}_z)$$

$$= \dot{R}\hat{e}_r + R\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z$$

$\dot{R} = 0$ $\quad \tan(\alpha) = \frac{z}{R\theta}$

$z = R\theta\tan\alpha$

$\dot{z} = R\dot{\theta}\tan\alpha$

So

$$\vec{V} = R\dot{\theta}\hat{e}_\theta + R\dot{\theta}\tan\alpha\,\hat{e}_z$$

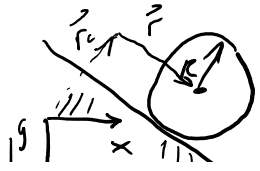
or $\vec{V} = \left(R\hat{e}_\theta + R\tan\alpha\,e_z\right)\dot{\theta}$

or $\bar{V} = \begin{Bmatrix} 0 \\ R \\ R\tan\alpha \end{Bmatrix}\dot{\theta}$

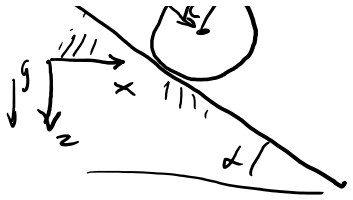
$\bar{J}$

2-17 A disc rolls down a hill as shown:

T+ start from rest @



a hill as shown:
It starts from rest @ height = z, find the velocity @ height 0
i) ignoring disc moment of inertia
ii) including moment of inertia



solution: use conservation of Energy (all energy conserved)

i) $E_1 = mgz$ @ $z = z$, $\dot{z} = 0$

$E_2 = \frac{1}{2}mv^2$ @ $z = 0$

$= \frac{1}{2}mr^2\dot{\theta}^2$

$\vec{v} = \frac{d}{dt}(\vec{r}_c + \vec{r}) = r\dot{\theta}\hat{r}$ ← added a constant vector

$\vec{v} = r\dot{\theta}(\cos\alpha\,\hat{x} + \sin\alpha\,\hat{y})$

$E_1 = E_2$

$mgz = \frac{1}{2}mv^2 \rightarrow \underline{v = \sqrt{2gz}}$

ii) $E_1 = mgz$

$E_2 = \frac{1}{2}mv^2 + \frac{1}{2}I\dot{\theta}^2$; $I = \frac{1}{2}mr^2$

$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\theta}^2 = \frac{3}{4}mr^2\dot{\theta}^2$

$E_1 = E_2$

$mgz = \frac{3}{4}mr^2\dot{\theta}^2 \rightarrow$ & $|v| = r\dot{\theta} = v$

$mgz = \frac{3}{4}mv^2$

$v = \sqrt{\frac{4}{3}gz}$