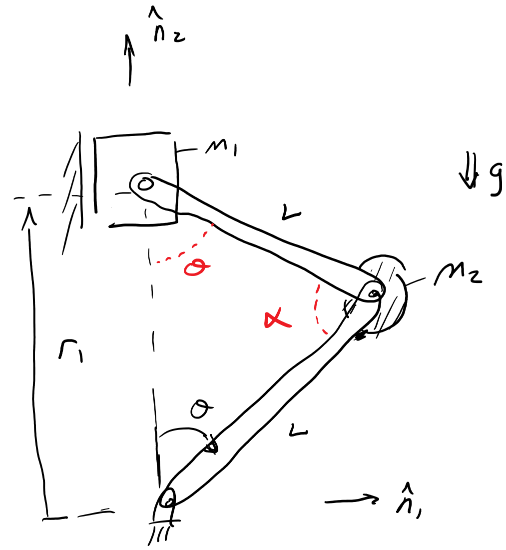
Chap. 4 HW solutions 5, 7

Friday, July 21, 2017 9:07 AM

Consider the four-bar mechanism -RRRP- with mass m1 the vertical slider and mass m2 located at the end of the link connected with a revolute to ground. Find the equations of motion with theta the generalized coordinate (Use the method of Lagrange)



$q = \{\theta\}$, 1 dof system.

$T = \frac{1}{2} m_1 \vec{v}_1 \cdot \vec{v}_1 + \frac{1}{2} m_2 \vec{v}_2 \cdot \vec{v}_2$

& $V = m_1 g\, \hat{r}_1 \cdot \hat{n}_2 + m_2 g\, \hat{r}_2 \cdot \hat{n}_2$

need $\vec{r}_1$, $\vec{r}_2$ as a function of θ.

$\vec{r}_1 = r_1 \hat{n}_2$, from symmetry: $\alpha = 180 - 2\theta$

From law of cosines: $r_1 = \pm\sqrt{2L^2 - 2L^2 \cos(180-2\theta)}$

$r_1 = \pm L\sqrt{2(1+\cos 2\theta)} = \pm 2L\sqrt{\cos(\theta)^2}$

use + sol'n only $= \pm 2L\cos\theta = r_1$

* in general, this solution is more complex,

$\vec{v}_1 = -2L s\theta\, \dot{\theta}\, \hat{n}_2$

$\vec{r}_2 = L s\theta\, \hat{n}_1 + L c\theta\, \hat{n}_2$, $\vec{v}_2 = L c\theta \dot{\theta}\, \hat{n}_1 - L s\theta \dot{\theta}\, \hat{n}_2$

$T = \frac{1}{2} m_1 (4L^2 s^2\theta\, \dot{\theta}^2) + \frac{1}{2} m_2 (L^2 c^2\theta\, \dot{\theta}^2 + L^2 s^2\theta\, \dot{\theta}^2)$

$= 2 m_1 L^2 s^2\theta\, \dot{\theta}^2 + \frac{1}{2} m_2 L^2 \dot{\theta}^2$

$V = m_1 g\, 2L c\theta + m_2 g L c\theta$

$q = \theta$:

$\partial T / \partial \theta = 4 m_1 L^2 s\theta c\theta\, \dot{\theta}^2$

$\partial T / \partial \dot{\theta} = 4 m_1 L^2 s^2\theta\, \dot{\theta} + m_2 L^2 \dot{\theta}$

$\frac{d}{dt}(\partial T / \partial \dot{\theta}) = 4 m_1 L^2 \cdot 2 s\theta c\theta\, \dot{\theta}^2 + 4 m_1 L^2 s^2\theta\, \ddot{\theta} + m_2 L^2 \ddot{\theta}$

$\frac{\partial V}{\partial \theta} = -m_1 g\, 2L s\theta - m_2 g L s\theta$

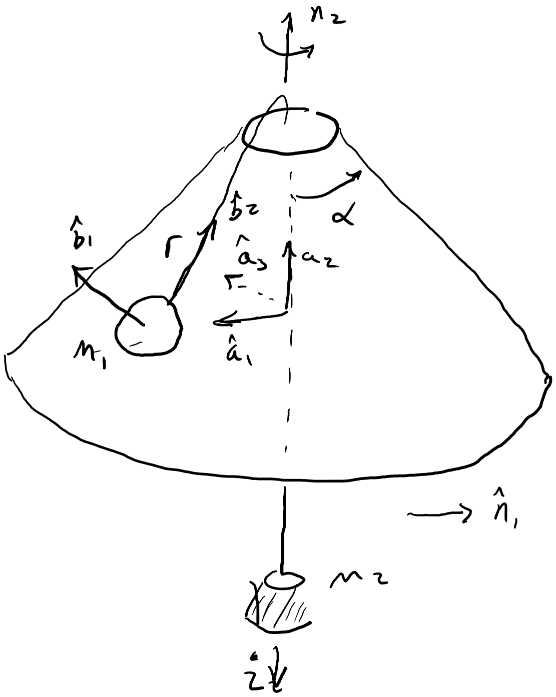
$Q_\theta = 0$

$(4 m_1 L^2 s^2\theta + m_2 L^2)\ddot{\theta} + 8 m_1 L^2 s\theta c\theta\, \dot{\theta}^2 - 4 m_1 L^2 s\theta c\theta\, \dot{\theta}^2$

$- (2m_1 + m_2) g L s\theta = 0$

or $\boxed{(4 m_1 s^2\theta + m_2)L^2\ddot{\theta} + 4 m_1 L^2 s\theta c\theta\, \dot{\theta}^2 - (2m_1+m_2)\, g L s\theta = 0}$

<u>EoM</u>



1st, $R_{n_2,\theta}$; 2nd $R_{a_3,\alpha}$

$= \underline{R}^n_a \qquad = \underline{R}^a_b = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\vec{q} = \{r, \theta\}^T$

Kinematic constraint:

$\dot{z} = \vec{V}_1 \cdot \hat{b}_2$ & $\vec{V}_1 = f(r,\theta)$

$\vec{r}_1 = -r\hat{b}_2 \qquad {}^n\vec{\omega}^b = \dot{\theta}\hat{a}_2 = {}^b\begin{Bmatrix} s\alpha\dot{\theta} \\ c\alpha\dot{\theta} \\ 0 \end{Bmatrix}$

$\vec{V}_1 = -\dot{r}\hat{b}_2 + \begin{Bmatrix} s\alpha\dot{\theta} \\ c\alpha\dot{\theta} \\ 0 \end{Bmatrix} \times \begin{Bmatrix} 0 \\ -r \\ 0 \end{Bmatrix}$

$\vec{V}_1 = {}^b\begin{Bmatrix} 0 \\ -\dot{r} \\ -r s\alpha\dot{\theta} \end{Bmatrix}, \quad \vec{V}_2 = -\dot{z}\,\hat{n}_2 = +\dot{r}\,\hat{n}_2$

$T = \frac{1}{2} m_1 \vec{V}_1 \cdot \vec{V}_1 + \frac{1}{2} m_2 \vec{V}_2 \cdot \vec{V}_2$

$T = \frac{1}{2} m_1 \left(\dot{r}^2 + r^2 s^2\alpha\, \dot{\theta}^2\right) + \frac{1}{2} m_2 \dot{r}^2$

$V = m_1 g(-c\alpha\, r) + m_2 g(L - r)$

$z = L - r$ (L = total string length)

$\vec{r}_1 = -r\hat{b}_2 = {}^a\begin{Bmatrix} +s\alpha\, r \\ -c\alpha\, r \\ 0 \end{Bmatrix}$

& gravity in $-\hat{a}_2$

$q = r$

$\partial T/\partial r = m_1 r s^2\alpha\, \dot{\theta}^2$

$\partial T/\partial \dot{r} = m_1 \dot{r} + m_2 \dot{r}$

$q = \theta$

$\partial T/\partial \theta = 0$

$\partial T/\partial \dot{\theta} = m_1 r^2 s^2\alpha\, \dot{\theta}$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right) = (m_1 + m_2)\ddot{r}$$

$$\frac{\partial V}{\partial r} = -m_1 g c\alpha - m_2 g$$

$$Q_r = 0$$

$$(m_1 + m_2)\ddot{r} - m_1 r s^2\alpha \dot{\theta}^2 - (m_1 c\alpha + m_2)g = 0$$

EOM 1

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = m_1 r^2 s^2\alpha \ddot{\theta} + 2 m_1 r s^2\alpha \dot{\theta}\dot{r}$$

$$\frac{\partial V}{\partial \theta} = 0$$

$$Q_\theta = 0$$

$$m_1 r^2 s^2\alpha \ddot{\theta} + 2 m_1 r s^2\alpha \dot{\theta}\dot{r} = 0$$

EOM 2