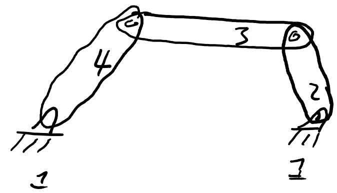
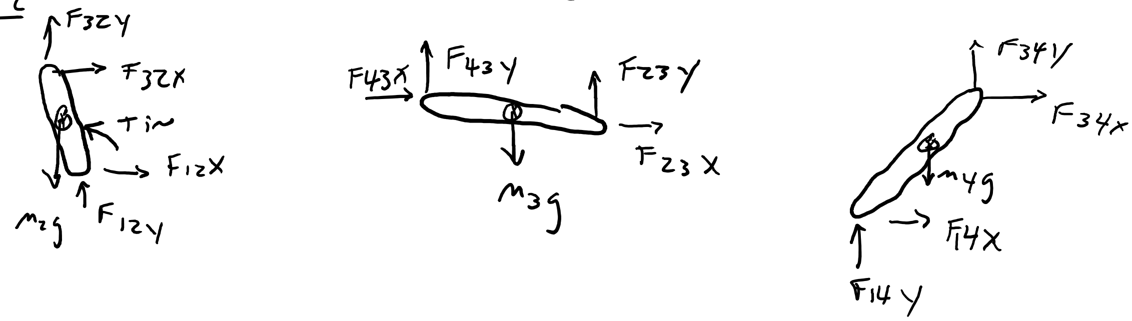
Chap 2 HW:

Monday, June 12, 2017 3:27 PM

2-2 Create a FBD for the 4-bar



FBD 2



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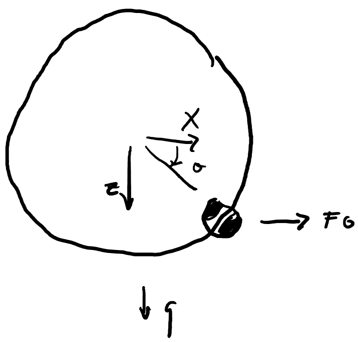


$$\vec{v} = \frac{d}{dt}(\vec{r}) = \frac{d}{dt}\left(R\,c\theta\,\hat{x} + R\,s\theta\,\hat{y}\right)$$

$$\vec{V} = \left(-R\,s\theta\,\hat{x} + R\,c\theta\,\hat{y}\right)\dot{\theta}$$

or $\bar{V} = \begin{Bmatrix} -R s\theta \\ R c\theta \end{Bmatrix} \dot{\theta}$

$\uparrow J = \frac{\partial r}{\partial q}.$

& $J^T \vec{F} = 0$ under static equilibrium

$$\vec{F} = F_o\,\hat{x} + mg\,\hat{z}$$

$$\begin{bmatrix} -R s\theta & R c\theta \end{bmatrix} \begin{Bmatrix} F_0 \\ mg \end{Bmatrix} = 0$$

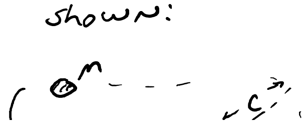
$$-R s\theta F_0 + R c\theta mg = 0$$

$$s\theta F_0 = mg\, c\theta$$

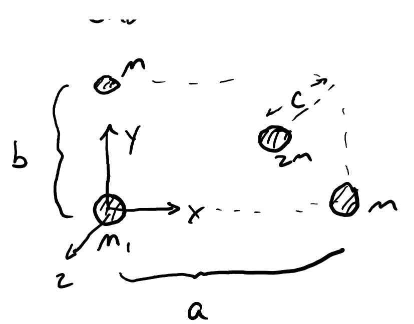
or $t\theta = \frac{mg}{F_0}$

$$\theta = \text{atan}\left(\frac{mg}{F_0}\right)$$

2-8 4 particles, masses $\{m, m, m, 3m\}$ are located as shown:



i) find cm., ii) find locations from cm to each particle



i) find cm., ii) find locations from cm to each particle $\vec{\rho}_i$

iii) sketch in Matlab

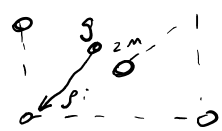
i): $\dfrac{\int r\,dm}{\int dm}$ or $\dfrac{\Sigma r_i m_i}{\Sigma m_i}$ for discrete system.

$$x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m_i} = \frac{(m+2m)a}{5m} = \frac{3}{5}a$$

$$y_{cm} = \frac{\Sigma y_i m_i}{\Sigma m_i} = \frac{(m+2m)b}{5m} = \frac{3}{5}b$$

$$z_{cm} = \frac{\Sigma z_i m_i}{\Sigma m_i} = \frac{2mc}{5m} = \frac{2}{5}c$$

$\bar{r}_i = \bar{r}_g + \bar{\rho}_i$ w/ $\vec{r}_g = \{\frac{3}{5}a, \frac{3}{5}b, \frac{2}{5}c\}$



$\bar{\rho}_i = \bar{r}_i - \bar{r}_g$

$$\rho_1 = \vec{0} - \vec{r}_g = \{-\tfrac{3}{5}a, -\tfrac{3}{5}b, -\tfrac{2}{5}c\}^T$$

$$\rho_2 = [a,0,0] - \vec{r}_g = \{\tfrac{2}{5}a, -3/5\,b, -2/5\,c\}^T$$

$$\rho_3 = (0,b,0) - \vec{r}_g = \{-3/5\,a, 2/5\,b, -2/5\,c\}^T$$

$$\rho_4 = (a,b,c) - \vec{r}_g = \{\tfrac{2}{5}a, 2/5\,b, 3/5\,c\}^T$$

in matlb:

define $\vec{r}_i$, $\vec{r}_g$,

Then >>rho_1 = r_1 - r_g;

>> rho_2 = r_2 - r_g;

etc..

& >>plot3(rho_1, 'ro'); ← my choice of marker & color

>> hold on

>> plot 3(rho_2, 'rx');

etc...

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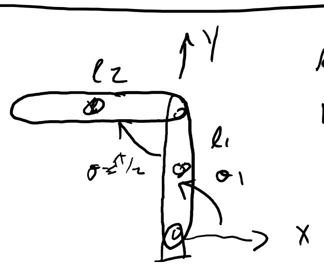
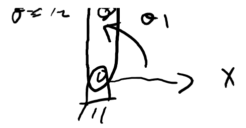


Robot link w/ θ fixed, find velocity when $\theta_1 = 180°$. (starts @ rest)

Idealized (no energy loss) conditions



Idealized (no energy loss) conditions

solution: assume constant energy, $\vec{V}^2 = |\vec{v}|^2 = \bar{v}^\dagger v$

$E_1 = E_2$

$\theta = $ constant so $\dot{\theta}_1 = \dot{\theta}_2$

$$E_1 = m_1 g\, l_1/2 + m_2 g\, l_1$$

$$E_2 = \tfrac{1}{2} m_1 \bar{V}_1^2 + \tfrac{1}{2} I_1 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 \bar{V}_2^2 + \tfrac{1}{2} I_2 \dot{\theta}_2^2$$

$$\vec{V}_1 = \tfrac{d}{dt}\left(l_1/2\, e^{i\theta_1}\right) \qquad \vec{V}_2 = \tfrac{d}{dt}\left(l_1 e^{i\theta_1} + l_2/2\, e^{i\theta_1 + \frac{\pi}{2}}\right)$$

$$\vec{V}_1 = l_1/2\, \dot{\theta}_1 i e^{i\theta_1} \qquad \vec{V}_2 = l_1 \dot{\theta}_1 i e^{i\theta_1} + l_2/2\, \dot{\theta}_1 i e^{i\theta_1 + \frac{\pi}{2}}$$

* when dotting vectors in Polar coord's, multiply magnitude, & ~~take~~ cos (angle difference)

ex: $\vec{v}_1 \cdot \vec{v}_1 = l_1/2\, \dot{\theta}_1 e^{i(\theta_1+\pi/2)} \cdot l_1/2\, \dot{\theta}_1 e^{i(\theta_1+\pi/2)}$

$$\left(l_1/2\, \dot{\theta}_1\right)^2 \cos\left(\theta_1 + \pi/2 - \theta_1 + \pi/2\right)$$

$$\vec{V}_1^2 = l_1/2\, \dot{\theta}_1^2$$

$$\vec{V}_2 \cdot \vec{V}_2 = \left(l_1 \dot{\theta}_1 i e^{i\theta_1} + \tfrac{l_2}{2} \dot{\theta}_1 i e^{i(\theta_1+\pi/2)}\right) \cdot \left(l_1 \dot{\theta}_1 i e^{i\theta_1} + \tfrac{l_2}{2} \dot{\theta}_1 i e^{i(\theta_1+\pi/2)}\right)$$

$$= l_1^2 \dot{\theta}_1^2 \overbrace{\cos(0)}^{1} + l_1 \dot{\theta}_1\, l_2/2\, \dot{\theta}_1 \overbrace{\cos(-90)}^{0} + \tfrac{l_2}{2} \dot{\theta}_1\, l_1 \dot{\theta}_1 \overbrace{\cos(90)}^{0} + \left(l_2/2\, \dot{\theta}_1\right)^2 \overbrace{\cos(0)}^{1}$$

$$\vec{V}_2^2 = l_1^2 \dot{\theta}_1^2 + \left(l_2/2\right)^2 \dot{\theta}_1^2 = \left(l_1^2 + l_2^2/4\right) \dot{\theta}_1^2$$

$$I = \tfrac{1}{12} m l^2$$

$$E_2 = \tfrac{1}{2}\left(m_1 \tfrac{l_1^2}{2} \dot{\theta}_1^2 + \tfrac{1}{12} m_1 l_1^2 \dot{\theta}_1^2 + m_2\left(l_1^2 + \tfrac{l_2^2}{4}\right)\dot{\theta}_1^2 + \tfrac{1}{12} m_2 l_2^2 \dot{\theta}_1^2\right)$$

$$= \tfrac{1}{2} \dot{\theta}_1^2 \left(\tfrac{7}{12} m_1 l_1^2 + m_2 l_1^2 + m_2 \tfrac{4}{12} l_2^2\right)$$

$E_1 = E_2$

$$\left(\tfrac{m_1}{2} + m_2\right) g\, l_1 = \tfrac{1}{2} \dot{\theta}_1^2 \left(\tfrac{7}{12} m_1 l_1^2 + m_2 \left(l_1^2 + \tfrac{1}{3} l_2^2\right)\right)$$

$$\dot{\theta}_1 = \left[\frac{2\left(m_1/2 + m_2\right) g\, l_1}{7/12\, m_1 l_1^2 + m_2\left(l_1^2 + \frac{1}{3} l_2^2\right)} \right]^{1/2}$$