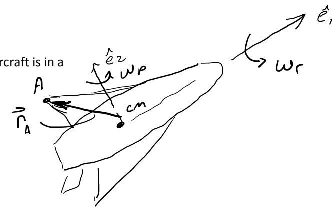
Ch3 practice

Monday, June 26, 2017 1:01 PM

3.1: Consider the aircraft shown, find the angular velocity of the wing tip when the aircraft is in a roll with rate w_r, pitch of rate w_p, and linear forward speed of v_1



$$\vec{V}_A = \overset{v_1}{\vec{V}_{cm}} + \frac{d}{dt}(\vec{r}_A) \; ; \quad \vec{r}_A \text{ is constant length so:}$$

$$\frac{d}{dt}(\vec{r}_A) = {}^{e}\frac{d}{dt}\vec{r}_A{}^{e} + {}^{n}\vec{\omega}^{e} \times \vec{r}_A \; ;$$

$$\text{\& let } \vec{r}_A = r_1\hat{e}_1 + r_2\hat{e}_2 \, ,$$

$${}^{n}\omega^{e} = \omega_r\hat{e}_1 + \vec{\omega}_p\hat{e}_2$$

$$\vec{V}_A = \vec{V}_{cm} + {}^{e}\begin{Bmatrix}\omega_r\\ \omega_p\\ 0\end{Bmatrix} \times {}^{e}\begin{Bmatrix}r_1\\ r_2\\ 0\end{Bmatrix} \; ; \quad V_{cm} = V_1\hat{e}_1$$

$$\vec{V}_A = V_1\hat{e}_1 + {}^{e}\begin{Bmatrix}0\\ 0\\ \omega_r r_2 - \omega_p r_1\end{Bmatrix} = {}^{e}\begin{Bmatrix}V_1\\ 0\\ \omega_r r_2 - \omega_p r_1\end{Bmatrix}$$

in body-fixed $\{e\}$ coords.

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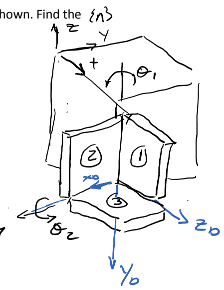
3.5 A stowed solar panel on the satellite undergoes two rotations in the sequence shown. Find the orientation of the panel relative to the satellite frame.



Rotations are:

R_{z,θ_1}, then about the new $\hat{x}$,

R_{x,θ_2}:

For moving-body rotations:

$$\underline{R}^{n}_{b} = R_{z,\theta_1} * R_{x,\theta_2} = \begin{bmatrix} c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix} * \begin{bmatrix} 1 & 0 & 0\\ 0 & c_2 & -s_2\\ 0 & s_2 & c_2\end{bmatrix}$$

$$\underline{R}^{n}_{b} = \begin{bmatrix} c_1 & -s_1c_2 & +s_1s_2\\ s_1 & c_1c_2 & -c_1s_2\\ 0 & s_2 & c_2\end{bmatrix}$$

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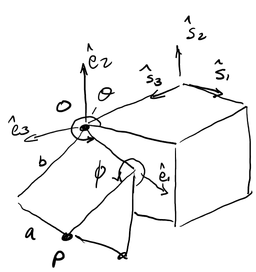
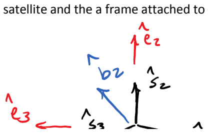
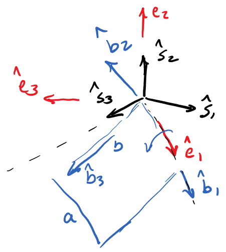
3.10: A compound solar panel on a satellite is shown. The inner panel undergoes a rotation about the vertical axis, e2, while the outer panel undergoes a rotation about the e1 axis.

a) Find the velocity and acceleration of point P on the edge assuming the angular rotation rates are constant.
b) Describe these in the a frame attached to the satellite and the a frame attached to the outer panel.







a) Find $\vec{V}_P, \vec{a}_P$ relative to satellite body (0):

$$\vec{V}_P = \frac{d}{dt}(\vec{r}) = \frac{d}{dt}\left(a\hat{e}_1 + b\hat{b}_3\right)$$

$$\vec{V}_P = 0 + {}^{n}\omega^{e} \times a\hat{e}_1 + {}^{n}\omega^{b} \times b\hat{b}_3$$

Need ω's, Rotations:

$$\underline{R}^s_e = R_{\theta,2} = \begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix} \qquad R^e_b = R_{\phi,1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$${}^{n}\omega^{e} = \dot{\theta}\hat{e}_2, \quad {}^{n}\omega^{b} = {}^{n}\omega^{e} + {}^{e}\omega^{b} = \dot{\theta}\hat{e}_2 + \dot{\phi}\hat{b}_1 \quad \leftarrow \text{mult. by } R_b^{e\,T}$$

$$= \dot{\theta}c\phi\hat{b}_2 - \dot{\theta}s\phi\hat{b}_3 + \dot{\phi}\hat{b}_1$$

$${}^{n}\omega^{b} = \{\dot{\phi}, \dot{\theta}c\phi, -\dot{\theta}s\phi\}^T$$

$$\vec{V}_P = \dot{\theta}\hat{e}_2 \times a\hat{e}_1 + {}^{b}\begin{pmatrix} \dot{\phi} \\ \dot{\theta}c\phi \\ -\dot{\theta}s\phi \end{pmatrix} \times {}^{b}\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix}$$

$$= -\dot{\theta}a\hat{e}_3 + {}^{b}\begin{pmatrix} \dot{\theta}c\phi b \\ b\dot{\phi} \\ 0 \end{pmatrix} =$$

$$= R_b^{e\,T} \cdot -\dot{\theta}a\hat{e}_3 + \begin{pmatrix} \dot{\theta}c\phi b \\ b\dot{\phi} \\ 0 \end{pmatrix} = \begin{pmatrix} b\dot{\theta}c\phi \\ b\dot{\phi} - \dot{\theta}as\phi \\ -\dot{\theta}ac\phi \end{pmatrix} = \vec{V}_P$$

$$\vec{a}_P = \frac{d}{dt}\vec{V}_P = {}^{b}\frac{d}{dt}(\vec{V}_P) + {}^{n}\omega^{b} \times \vec{V}_P$$

$${}^{b}\begin{pmatrix} b\ddot{\theta}c\phi - b\dot{\theta}\dot{\phi}s\phi \\ b\ddot{\phi} - \ddot{\theta}as\phi - a\dot{\theta}\dot{\phi}c\phi \\ -a\ddot{\theta}c\phi + a\dot{\theta}\dot{\phi}s\phi \end{pmatrix} + {}^{b}\begin{pmatrix} \dot{\phi} \\ \dot{\theta}c\phi \\ -\dot{\theta}s\phi \end{pmatrix} \times \begin{pmatrix} b\dot{\theta}c\phi \\ b\dot{\phi} - a\dot{\theta}s\phi \\ -a\dot{\theta}c\phi \end{pmatrix}$$

$$\text{"} \quad + {}^{b}\begin{pmatrix} -\dot{\theta}c\phi a\dot{\theta}c\phi + \dot{\theta}s\phi b\dot{\phi} - \dot{\theta}s\phi a\dot{\theta}s\phi \\ -\dot{\theta}s\phi b\dot{\theta}c\phi + \dot{\phi}a\dot{\theta}c\phi \\ \dot{\phi}^2 b - \dot{\phi}\dot{\theta}as\phi - \dot{\theta}^2 bc^2\phi \end{pmatrix}$$

$$\vec{a}_P = \begin{pmatrix} b\ddot{\theta}c\phi - \cancel{b\dot{\theta}\dot{\phi}s\phi} - a\dot{\theta}^2c^2\phi + \cancel{b\dot{\theta}\dot{\phi}s\phi} - \dot{\theta}^2s^2\phi\, a \\ b\ddot{\phi} - \ddot{\theta}as\phi - \cancel{a\dot{\theta}\dot{\phi}c\phi} - b\dot{\theta}^2s\phi c\phi + \cancel{a\dot{\theta}\dot{\phi}c\phi} \\ -a\ddot{\theta}c\phi + \cancel{a\dot{\theta}\dot{\phi}s\phi} + \dot{\phi}^2b - \cancel{a\dot{\theta}\dot{\phi}s\phi} - \dot{\theta}^2bc^2\phi \end{pmatrix} = \begin{pmatrix} b\ddot{\theta}c\phi - \dot{\theta}^2a \\ b\ddot{\phi} - a\ddot{\theta}s\phi - b\dot{\theta}^2s\phi c\phi \\ -a\ddot{\theta}c\phi + \dot{\phi}^2b - b\dot{\theta}^2c^2\phi \end{pmatrix}$$

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