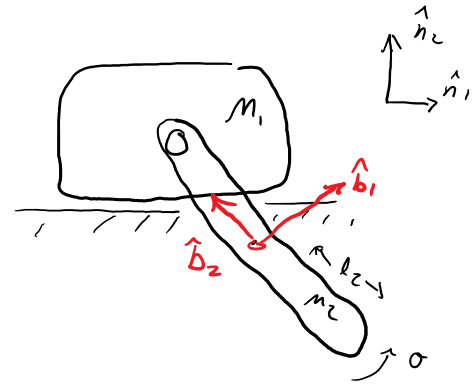
Chap 5 Practice: 5.16, 5.17

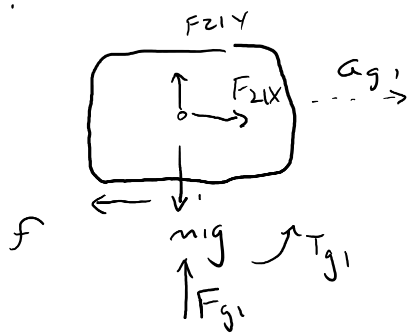
Sunday, July 2, 2017 9:50 AM

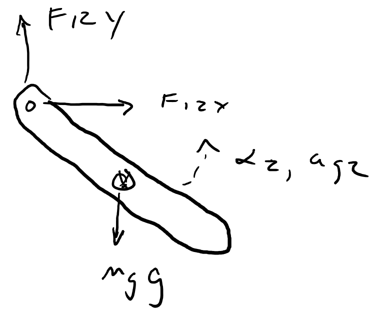
5.16 Find the EOMs for the block w/ pendulum when the block slips w/ Coulomb friction.



$$R^n_b = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2 bodies:





UK's, $\underbrace{F_{21x}\ F_{21y}\ \ F_{12x}\ F_{12y}}_{\text{equal \& opp.}}$; F_{g1}, T_{g1}, f, a_{g1x}, α_2 (9)

eq's 6 eom + 2 equal & opp + 1 coulomb law

Kinematics:

$\vec{r}_1 = x\,\hat{n}_1$, $v_1 = \dot{x}\,n_1$, $\vec{a}_{g1} = \ddot{x}\,\hat{n}_1$ | ${}^n\omega^b = \dot{\theta}\,\hat{b}_3$

$\vec{r}_{g2} = x\hat{n}_1 - l/2\,\hat{b}_2$; $\vec{V}_{g2} = \dot{x}\hat{n}_1 - l/2\,\dot{\theta}\,\hat{b}_1$ | ${}^n\alpha^b = \ddot{\theta}\,\hat{b}_3$

$$\vec{a}_{g2} = \ddot{x}\,\hat{n}_1 - l/2\,\ddot{\theta}\,\hat{b}_1 + l/2\,\dot{\theta}^2\,\hat{b}_2$$

$$\vec{a}_{g2} = {}^b\begin{pmatrix} \ddot{x}\,c\theta - l/2\,\ddot{\theta} \\ -\ddot{x}\,s\theta + l/2\,\dot{\theta}^2 \\ 0 \end{pmatrix}$$

EOM: ...ion opposes V_{g1} dir.

EOM:

sign opposes V_{g1} dir.

① $F_{21x} - \mu N_{g1} = m\ddot{x}$

② $F_{21y} + N_{g1} - m_1 g = 0$

③ $T_{g1} = 0$

let $F_{12x} = -F_{21x}$ etc.

$$\underbrace{\begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_b^{n\,T}} \begin{pmatrix} F_{12x} \\ F_{12y} \\ 0 \end{pmatrix} = \begin{pmatrix} c\theta F_{12x} + s\theta F_{12y} \\ -s\theta F_{12x} + c\theta F_{12y} \\ 0 \end{pmatrix}$$

④ $-c\theta F_{21x} - s\theta F_{21y} = m_2(\ddot{x}\, c\theta - \frac{l}{2}\ddot{\theta})$ $\quad - s\theta m g$

⑤ $+ s\theta F_{21x} - c\theta F_{21y} = m_2(-\ddot{x}\, s\theta + \frac{l}{2}\dot{\theta}^2)$ $\quad - c\theta m g$

⑥ $-(+s\theta F_{21x} - c\theta F_{21y})\frac{l}{2} = I_2 \ddot{\theta}$ $\quad I_2 = \frac{1}{12} m l^2$

$$\begin{bmatrix} 1 & 0 & 0 & -\mu & -m_1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -c\theta & -s\theta & 0 & 0 & -m_2 c\theta & +m_2\frac{l}{2} \\ s\theta & -c\theta & 0 & 0 & +m_2 s\theta & 0 \\ -s\theta\frac{l}{2} & +c\theta\frac{l}{2} & 0 & 0 & 0 & -\frac{1}{12}ml^2 \end{bmatrix} \begin{pmatrix} F_{21x} \\ F_{21y} \\ T_{g1} \\ N_{g1} \\ \ddot{x} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} 0 \\ mg \\ 0 \\ +s\theta m g \\ +m_2\frac{l}{2}\dot{\theta}^2 + c\theta m g \\ 0 \end{pmatrix}$$

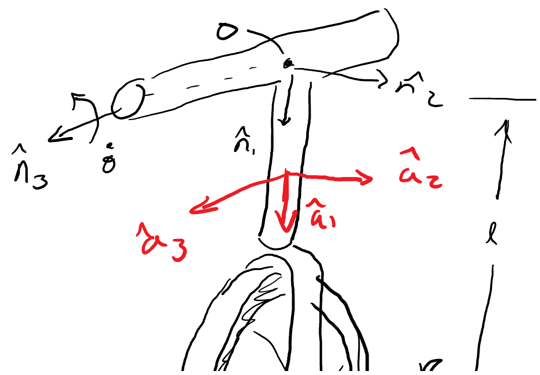
$$\underline{C}\,\vec{F} = \vec{b}$$

$$\vec{F} = \underline{C}^{-1}\vec{b}$$

5-17

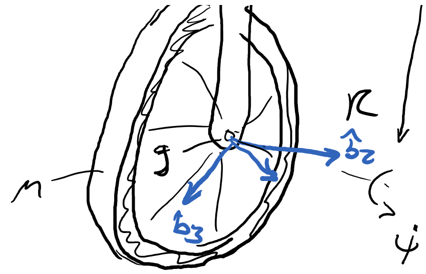
A wheel of mass m and radius R rotates about two axes with constant Rotation rates theta_dot, psi_do, calculate the required moment Necessary to maintain this motion. Ignore gravity

Find $\underline{M}_O$



$$\underline{R}_a^n = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad R_b^a = \begin{bmatrix} c\psi & 0 & s\psi \\ 0 & 1 & 0 \\ -s\psi & 0 & c\psi \end{bmatrix}$$

$$- \cdots \begin{bmatrix} \cdots \\ 0 \quad 0 \quad 1 \end{bmatrix}, \quad \cdots \begin{bmatrix} \cdots \\ -s\psi \quad 0 \quad c\psi \end{bmatrix}$$



★ M_O will have to account for both $\dot{\theta}$ & $\dot{\psi}$ motion. Note that an input about $\hat{b}_2$ would be needed.

First, kinematics!

$${}^n\vec{\omega}^b = \dot{\theta}\,\hat{a}_3 + \dot{\psi}\,\hat{b}_2$$

$${}^n\omega^b = \begin{pmatrix} -s\psi\dot{\theta} \\ \dot{\psi} \\ c\psi\dot{\theta} \end{pmatrix}; \quad {}^n\alpha^b = \begin{pmatrix} -\dot{\psi}\dot{\theta}c\psi - s\psi\ddot{\theta}^{\,0} \\ \ddot{\psi}^{\,0} \\ -\dot{\psi}\dot{\theta}s\psi + c\psi\ddot{\theta}^{\,0} \end{pmatrix}$$

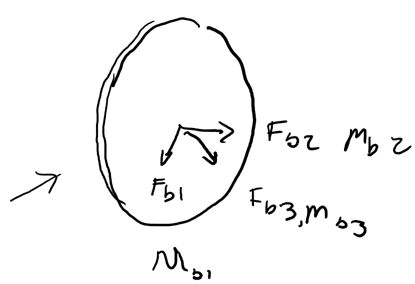
but $\dot{\psi} = \text{const.}$, $\dot{\theta} = \text{const}$

$$\vec{V}_g = \frac{d}{dt}(\vec{r}_g) = \frac{d}{dt}(l\,\hat{a}_1) = \dot{\theta}\,\hat{a}_3 \times l\,\hat{a}_1 = \dot{\theta}\,l\,\hat{a}_2$$

$$\vec{a}_g = \ddot{\theta}\,l\,\hat{a}_2 - \dot{\theta}^2 l\,\hat{a}_1 \rightarrow \underbrace{\begin{bmatrix} c\psi & 0 & -s\psi \\ 0 & 1 & 0 \\ s\psi & 0 & c\psi \end{bmatrix}}_{\leftarrow R_b^{a^T}} \begin{pmatrix} -\dot{\theta}^2 l \\ \ddot{\theta} l \\ 0 \end{pmatrix} =$$

$$\vec{a}_g = \begin{pmatrix} -c\psi\dot{\theta}^2 l \\ \ddot{\theta}\,l \\ -s\psi\dot{\theta}^2 l \end{pmatrix}$$

FBD wheel:



$${}^b I_g = \begin{bmatrix} I_1 & & 0 \\ & I_2 & \\ 0 & & I_3 \end{bmatrix}$$

$I_3 = I_1$

$$F_{b1} = -m\,c\psi\,\dot{\theta}^2 l$$

$$F_{b2} = 0$$

$$F_{b3} = -m\,s\psi\,\dot{\theta}^2 l$$

$$M_{b1} = I_1\alpha_1 - (I_2 - I_3)\,\omega_2\omega_3$$

$$M_{b2} = I_2\,0 - \cancel{(I_3 - I_1)}^{\,0}\,\omega_3\omega_1$$

$$M_{b3} = I_3\alpha_3 - (I_1 - I_2)\,\omega_1\omega_2$$

$$^b\vec{M}_g = \begin{cases} M_{b1} = I_1(-\dot{\psi}\dot{\theta}c\psi) - (I_2 - I_3)\dot{\psi}\dot{\theta}c\psi \\ M_{b2} = 0 \\ M_{b3} = I_3(-\dot{\psi}\dot{\theta}s\psi) - (I_1 - I_2)(-s\psi\dot{\theta}\dot{\psi}) \end{cases}$$

put in {a} coordinates

$$\begin{bmatrix} c\psi & 0 & s\psi \\ 0 & 1 & 0 \\ -s\psi & 0 & c\psi \end{bmatrix} \begin{pmatrix} -mc\psi\dot{\theta}l \\ 0 \\ -ms\psi\dot{\theta}l \end{pmatrix} = \begin{pmatrix} -m\dot{\theta}l \\ 0 \\ 0 \end{pmatrix} = {}^aF_g$$

$$\& \quad {}^aM_g = \begin{bmatrix} c\psi M_{b1} + s\psi M_{b3} \\ 0 \\ -s\psi M_{b1} + c\psi M_{b1} \end{bmatrix}$$

$$\hookrightarrow \quad {}^aM_o = l\hat{a}_1 \times {}^aF_g + {}^aM_g$$