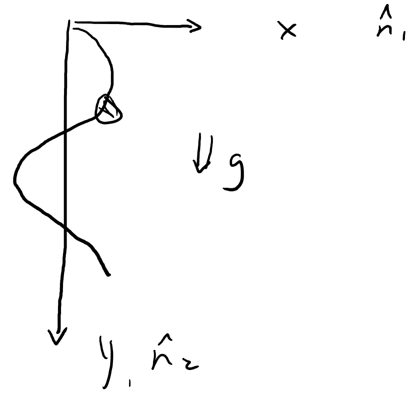
Ch. 4 Practice (1, 4) & HW (5, 7)

Thursday, July 20, 2017 11:50 AM



Find EOM

4.1 $x = A\sin\left(\frac{2\pi y}{L}\right)$

$q = \{y\}$

$\vec{r} = x\hat{n}_1 + y\hat{n}_2 = A s\left(\frac{2\pi y}{L}\right)\hat{n}_1 + y\hat{n}_2$

$\dot{\vec{r}} = A\cos\left(\frac{2\pi y}{L}\right)\cdot\frac{2\pi}{L}\dot{y}\,\hat{n}_1 + \dot{y}\,\hat{n}_2$

$T = \frac{1}{2}m\dot{\vec{r}}\cdot\dot{\vec{r}}$

$= \frac{1}{2}m\left(\left(\frac{2\pi A}{L}\right)^2\cos^2\left(\frac{2\pi y}{L}\right) + 1\right)\dot{y}^2$

$V = -mgy$

$q_1 = y$:

$\partial T/\partial y = -\frac{1}{2}m\dot{y}^2\left(\left(\frac{2\pi A}{L}\right)^2 2\cos\frac{2\pi y}{L}\sin\left(\frac{2\pi y}{L}\right)\right)$

$\frac{\partial T}{\partial \dot{y}} = m\left(\underbrace{\frac{2\pi A}{L}^2\cos^2\left(\frac{2\pi y}{L}\right)+1}_{\text{temp}}\right)\dot{y}$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) = m(\text{temp})\ddot{y}$

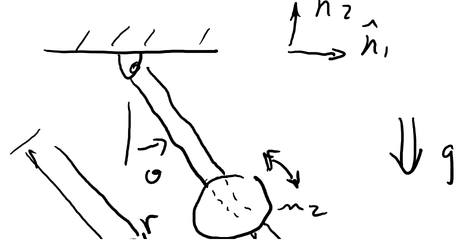
$\frac{\partial V}{\partial y} = -mg$

$Q_y = 0$

$$m\left(\frac{2\pi A}{L}^2\cos^2\left(\frac{2\pi y}{L}\right) + 1\right)\ddot{y} + m\dot{y}^2\left(\frac{2\pi A}{L}^2\cos\left(\frac{2\pi y}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\right) - mg = 0$$

EOM.

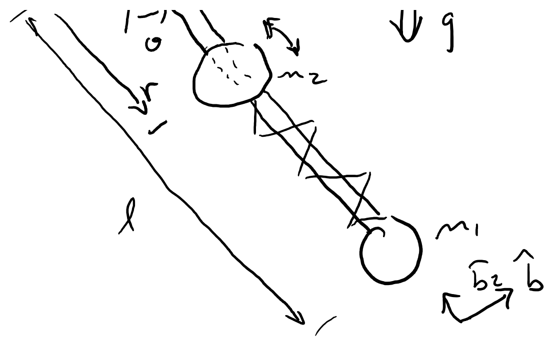
4-4



$q = \{r, \theta\}^T$

$T = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$

$q = (\cdot)$



$$T = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$\vec{r}_1 = -l\hat{b}_2 \;;\; \vec{v}_1 = l\dot{\theta}\hat{b}_1$$

$$\vec{r}_2 = -r\hat{b}_2 \;;\; \vec{v}_2 = -\dot{r}\hat{b}_2 + r\dot{\theta}\hat{b}_1$$

$$T = \tfrac{1}{2} m_1 l^2 \dot{\theta}^2 + \tfrac{1}{2} m_2 \left(r^2\dot{\theta}^2 + \dot{r}^2\right)$$

$$V = -m_1 l c\theta - m_2 r c\theta + \tfrac{1}{2}K(l-r)^2$$ ← assumes 0 unstretched length.

$q = r$:

$$\partial T/\partial r = m_2 r \dot{\theta}^2$$

$$\partial T/\partial \dot{r} = m_2 \dot{r}$$

$$\tfrac{d}{dt}\partial T/\partial \dot{r} = m_2 \ddot{r}$$

$$\frac{\partial V}{\partial r} = -m_2 c\theta - k(l-r)$$

$$Q_r = 0$$

$$\boxed{m_2\ddot{r} - m_2 r\dot{\theta}^2 - m_2 c\theta - k(l-r) = 0}$$

eom1

$q = \theta$

$$\partial T/\partial\theta = 0$$

$$\partial T/\partial\dot{\theta} = m_1(l^2 + r^2)\dot{\theta}$$

$$\tfrac{d}{dt}\left(\partial T/\partial\dot{\theta}\right) = m_1(l^2+r^2)\ddot{\theta} + 2m_1\dot{\theta} r\dot{r}$$

$$\frac{\partial V}{\partial\theta} = +m_1 l s\theta + m_2 r s\theta$$

$$Q_\theta = 0$$

$$\boxed{m_1(l^2+r^2)\ddot{\theta} + 2m_1\dot{\theta}\dot{r}r + m_1 l s\theta + m_2 r s\theta = 0}$$

eom2