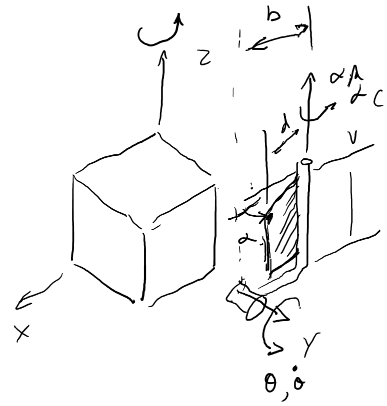
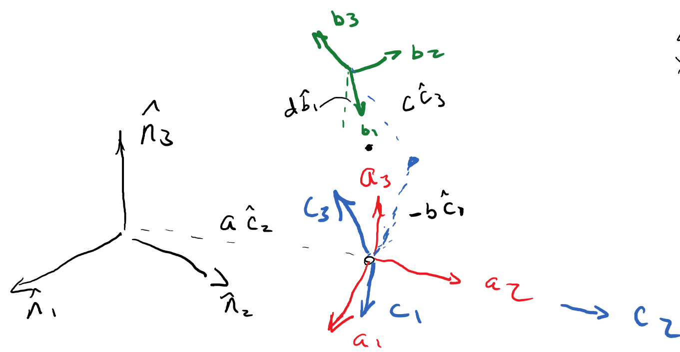
Practice 3.1, 3.5, 3.10   Homework 3.4, 3.6, 3.18

Friday, June 16, 2017   11:34 AM



3.4 A spacecraft spins about vertical axis e3 with constant rate cap_omega. A solar panel is deployed by rotation about two nonintersecting axes separated by a distance b.

a) Find the angular acceleration of the panel for arbitrary theta
b) Find the velocity and acceleration of point P for alpha_dot, theta_dot and cap_omega constant.
c) Describe the orientation of the solar panel relative to the spacecraft as a function of alpha and theta.



let Ω' be $\measuredangle$ associated w/ Ω

$$R^n_a = \begin{bmatrix} c\Omega' & -s\Omega' & 0 \\ s\Omega' & c\Omega' & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad R^a_c = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}; \quad R^c_b = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^n_b \qquad\qquad R^a_b = \begin{bmatrix} c\theta c\alpha & -c\theta s\alpha & s\theta \\ s\alpha & c\alpha & 0 \\ -s\theta c\alpha & +s\theta s\alpha & c\theta \end{bmatrix}$$

$${}^n\omega^b = {}^n\omega^a + {}^a\omega^c + {}^c\omega^b = \Omega a_3 + \dot\theta c_2 + \dot\alpha \hat b_3$$

$$= \begin{pmatrix} -s\theta c\alpha \Omega \\ s\theta s\alpha \Omega \\ c\theta \Omega \end{pmatrix} + \begin{pmatrix} s\alpha \dot\theta \\ c\alpha \dot\theta \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix}$$

$${}^n\alpha^b = \begin{Bmatrix} -\dot\theta\Omega\, c\theta c\alpha + \dot\alpha\Omega\, s\theta s\alpha + \dot\alpha\dot\theta c\alpha \\ +\dot\theta\Omega\, c\theta s\alpha + \dot\alpha\Omega\, s\theta c\alpha - \dot\alpha\dot\theta s\alpha \\ -\dot\theta\Omega\, s\theta + \ddot\alpha \end{Bmatrix}$$

$$r = a\hat c_2 - b\hat c_1 + c\hat c_3 + d b_1$$

$$V = \tfrac{d}{dt}(r) = 0 + {}^n\omega^c \times {}^c\begin{pmatrix} -b \\ +a \\ c \end{pmatrix} + {}^n\omega^b \times {}^b\begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix}$$

$${}^n\omega^c = {}^c\begin{pmatrix} -s\theta\Omega \\ \dot\theta \\ c\theta\Omega \end{pmatrix} \qquad {}^n\omega^b = \begin{pmatrix} -s\theta c\alpha\Omega + s\alpha\dot\theta \\ s\theta s\alpha\Omega + c\alpha\dot\theta \\ c\theta\Omega \end{pmatrix}$$

$$\vec{V} = {}^{c}\begin{Bmatrix} \dot\theta c - c\theta\Omega a \\ -c\theta\Omega b + s\theta\Omega c \\ -s\theta\Omega a + \dot\theta b \end{Bmatrix} + {}^{b}\begin{Bmatrix} 0 \\ c\theta\Omega d \\ + s\theta c\alpha\Omega d - s\alpha\dot\theta d \end{Bmatrix}$$

or

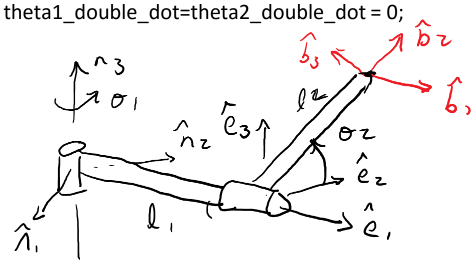
$$R_b^{c\,T} = \begin{bmatrix} c\alpha & s\alpha & 0 \\ -s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \\ \\ \end{Bmatrix}$$

$${}^{b}\vec{V} = \begin{Bmatrix} c\alpha(\dot\theta c - c\theta\Omega a) + s\alpha(-c\theta\Omega b + s\theta\Omega c) + 0 \\ -s\alpha(\dot\theta c - c\theta\Omega a) + c\alpha(-c\theta\Omega b + s\theta\Omega c) + c\theta\Omega d \\ -s\theta\Omega a + \dot\theta b + s\theta c\alpha\Omega d - s\alpha\dot\theta d \end{Bmatrix}$$

$$\vec{a} = {}^{b}\begin{Bmatrix} \dot\alpha(-s\alpha)(\dot\theta c - c\theta\Omega a) + c\alpha s\theta\dot\theta\Omega a + \dot\alpha c\alpha(-c\theta\Omega b + s\theta\Omega c) + s\alpha(\dot\theta s\theta\Omega b + \dot\theta c\theta\Omega c) \\ \dot\alpha(-c\alpha)(\dot\theta c - c\theta\Omega a) - s\alpha(\dot\theta s\theta\Omega a) - \dot\alpha s\alpha(-c\theta\Omega b + s\theta\Omega c) + c\alpha(\dot\theta s\theta\Omega b + \dot\theta c\theta\Omega c) - \dot\theta s\theta\Omega d \\ -\dot\theta c\theta\Omega a + \dot\theta c\theta c\alpha\Omega d - \dot\alpha s\theta s\alpha\Omega d - \dot\alpha\dot\theta c\alpha d \end{Bmatrix}$$

$$+ \begin{Bmatrix} -s\theta c\alpha\Omega + s\alpha\dot\theta \\ s\theta s\alpha\Omega + c\alpha\dot\theta \\ c\theta\Omega + \dot\alpha \end{Bmatrix} \times {}^{b}\vec{V}$$

3.6 The two-link serial mechanism rotates about two axes as shown. Find the velocity and acceleration for the end point B when theta1_double_dot=theta2_double_dot = 0;



$$R_e^n = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_b^e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{bmatrix}$$

$${}^{n}\bar{\omega}^{b} = \dot\theta_1 \hat{e}_3 + \dot\theta_2 \hat{b}_1$$

$$= {}^{b}\begin{Bmatrix} \dot\theta_2 \\ s_2\dot\theta_1 \\ c_2\dot\theta_1 \end{Bmatrix}$$

$$\begin{pmatrix} s_2\dot\theta_1 \\ c_2\dot\theta_1 \end{pmatrix}$$

$$^{n}\vec\alpha^{b} = \begin{pmatrix} \ddot\theta_2 \\ \dot\theta_1\dot\theta_2 c_2 + s_2\ddot\theta_1 \\ -\dot\theta_1\dot\theta_2 s_2 + c_2\ddot\theta_1 \end{pmatrix}$$

$$\vec r = l_1\hat e_1 + l_2\hat b_2$$

$$\vec V_p = \frac{d}{dt}(r) = 0 + \dot\theta_1\hat e_3 \times l_1\hat e_1 + {}^{b}\begin{pmatrix} \dot\theta_2 \\ s_2\dot\theta_1 \\ c_2\dot\theta_1 \end{pmatrix} \times {}^{b}\begin{pmatrix} 0 \\ l_2 \\ 0 \end{pmatrix}$$

$$= {}^{e}\begin{pmatrix} 0 \\ \dot\theta_1 l_1 \\ 0 \end{pmatrix} + {}^{b}\begin{pmatrix} -c_2\dot\theta_1 l_2 \\ 0 \\ \dot\theta_2 l_2 \end{pmatrix}$$

$$\vec A_p = \frac{d}{dt}\vec V_p = {}^{e}\begin{pmatrix} 0 \\ \ddot\theta_1 l_1 \\ 0 \end{pmatrix} + \dot\theta_1\hat e_3 \times \dot\theta_1 l_1\hat e_2 + {}^{b}\begin{pmatrix} \dot\theta_1\dot\theta_2 s_2 l_2 - c_2\ddot\theta_1 l_2 \\ 0 \\ \ddot\theta_2 l_2 \end{pmatrix} + {}^{b}\begin{pmatrix} \dot\theta_2 \\ s_2\dot\theta_1 \\ c_2\dot\theta_1 \end{pmatrix} \times {}^{b}\begin{pmatrix} \\ \\ \end{pmatrix}$$

$$= \overset{R_b^{e\,T}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{bmatrix}} {}^{e}\begin{pmatrix} -\dot\theta_1^2 l_1 \\ \ddot\theta_1 l_1 \\ 0 \end{pmatrix} + {}^{b}\begin{pmatrix} \dot\theta_1\dot\theta_2 s_2 l_2 - c_2\ddot\theta_1 l_2 + s_2\dot\theta_1\dot\theta_2 l_2 \\ 0 - c_2^2\dot\theta_1^2 l_2 - \dot\theta_2^2 l_2 \\ \ddot\theta_2 l_2 + s_2 c_2\dot\theta_1^2 l_2 \end{pmatrix}$$

$$A_p = \begin{pmatrix} -\dot\theta_1^2 l_1 + \dot\theta_1\dot\theta_2 s_2 l_2 - c_2\ddot\theta_1 l_2 + s_2\dot\theta_1\dot\theta_2 l_2 \\ c_2\ddot\theta_1 l_1 - c_2^2\dot\theta_1^2 l_2 - \dot\theta_2^2 l_2 \\ -s_2\ddot\theta_1 l_1 + \ddot\theta_2 l_2 + s_2 c_2\dot\theta_1^2 l_2 \end{pmatrix}$$

3.18 write a matlab program that will animate a kinematic model of the box falling off the ledge with constant angular velocity .1 rad/s

See matlab tutorial on 3D animation