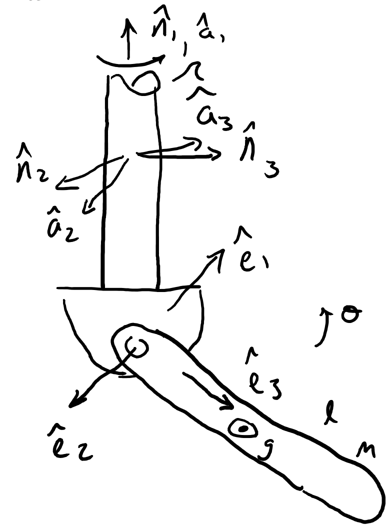
# Chap 5 Lagrange Rigid Body 5-24, 25, 28, 32

practice HW

Wednesday, July 26, 2017 6:13 PM

5-24 A thin rod with pin joint at one end Is spun about a vertical axis with constant angular velocity cap_omega. For generalized coordinate q = theta, solve the eom using Lagrange's equations



0) $q = \theta$

1) $T = \frac{1}{2} m \vec{V}_g \cdot \vec{V}_g + \frac{1}{2} \bar{\omega} \times I_g \bar{\omega}$

$\vec{r} = l/2\, \hat{e}_3$ , $\quad {}^{n}\bar{\omega}^{e} = \Omega \hat{a}_1 + \dot{\theta} \hat{e}_2$

$$R^{a}_{e} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$${}^{n}\bar{\omega}^{e} = {}^{e}\begin{pmatrix} c\theta\Omega \\ \dot{\theta} \\ s\theta\Omega \end{pmatrix}$$

$$\vec{V}_g = \begin{pmatrix} l/2\, \dot{\theta} \\ -l/2\, c\theta\Omega \\ 0 \end{pmatrix}, \qquad I = \frac{1}{12} m l^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \frac{1}{2} m \frac{l^2}{4} \left( \dot{\theta}^2 + c^2\theta\, \Omega^2 \right) + \frac{1}{2} \cdot \frac{1}{12} m l^2 \left( c^2\theta\, \Omega^2 + \dot{\theta}^2 \right)$$

$$V = -mg\, l/2\, c\theta$$

$q = \theta$:

$$\partial T / \partial \theta = m l^2/4 \left( -c\theta s\theta \Omega^2 \right) + \frac{m l^2}{12} \left( c\theta s\theta \Omega^2 \right)$$

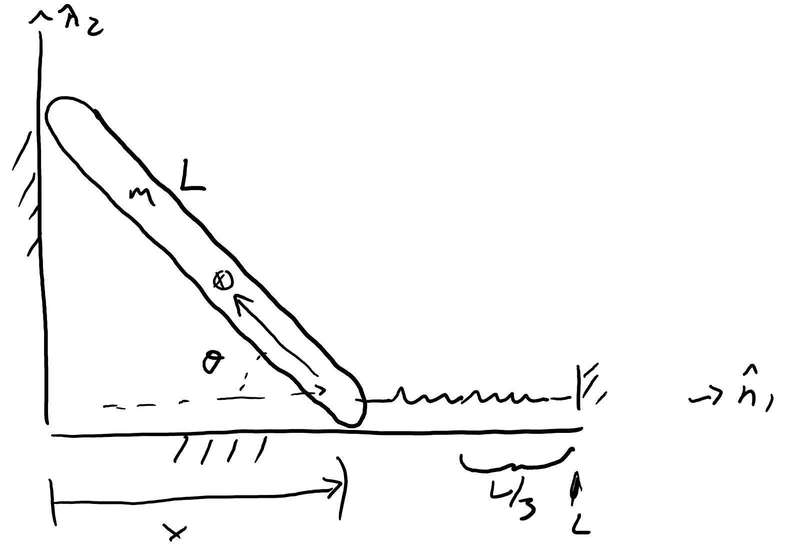
$$\partial T / \partial \dot{\theta} = m l^2/4\, \dot{\theta} + \frac{1}{12} m l^2 \dot{\theta} = \frac{m l^2}{3} \dot{\theta}$$

$$d/dt \left( \partial T / \partial \dot{\theta} \right) = m l^2/3\, \ddot{\theta}$$

$$\partial V / \partial \theta = mg\, l/2\, s\theta$$

$$\frac{m l^2}{3} \ddot{\theta} - \frac{m l^2}{6} c\theta s\theta \Omega^2 + mg\, l/2\, s\theta = 0$$

5-25: The thin rod is constrained with ends to lie along the horizontal and vertical walls as shown. The bottom end is attached to a linaer spring lying along the horizontal axis with unstretched length L/3. solve the equations of motion for generalized coordinates q =  using Lagrange. vb




(0) $q = x$

(1) $T = \frac{1}{2} m \vec{v}_g \cdot \vec{v}_g + \frac{1}{2} \vec{\omega} \cdot \underline{I}_g \vec{\omega}$

$V = mgh$

(1.5) $\vec{r} = \left(x - \frac{L}{2} c\theta\right) \hat{n}_1 + \frac{L}{2} s\theta \, \hat{n}_2$

$\&$ $x = L c\theta$

$\leftrightarrow \vec{r} = \frac{L}{2} c\theta \, \hat{n}_1 + \frac{L}{2} s\theta \, \hat{n}_2$

$\vec{V} = -\frac{L}{2} s\theta \dot{\theta} + \frac{L}{2} c\theta \dot{\theta}$

$I = \frac{1}{12} m L^2$

$T = \frac{1}{2} m \frac{L^2}{4} \dot{\theta}^2 + \frac{1}{2} \frac{1}{12} m L^2 \dot{\theta}^2 = \frac{1}{2} m \frac{L^2}{3} \dot{\theta}^2$

$V = mg \frac{L}{2} s\theta \quad + \frac{1}{2} k \left( x - \left(L - \frac{L}{3}\right) \right)$

$\underline{q = \theta:}$

$\partial T / \partial \theta = 0$

$\partial T / \partial \dot{\theta} = m \frac{L^2}{3} \dot{\theta}$

$\frac{d}{dt}(\quad) = m \frac{L^2}{3} \ddot{\theta}$

$\frac{\partial V}{\partial \theta} = -mg \frac{L}{2} c\theta$