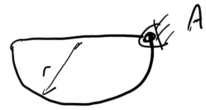
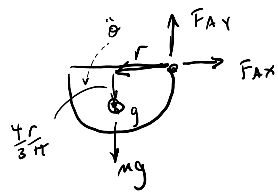
Basic examples: dynamics (review)

Wednesday, June 07, 2017 4:59 PM

A uniform half-disk of mass m and radius r is hinged freely at A.
The bar is released from rest when the top is horizontal. Find the force
Exerted on the bar by the pin at A at this instant.



solution: 1) FBD:



2) list uk's: $F_{Ax}, F_{Ay}, \ddot\theta, a_{gx}, a_{gy}$ (5)

3) eq's: 3 From EOM, need 2 more:

4) Kinematics

$$\vec a_g = \vec a_A + \frac{d^2}{dt^2}(\vec r)$$

$$\vec a_g = 0 + \ddot r \hat r + \vec\alpha\times\vec r + \vec\omega\times\vec\omega\times\vec r + 2\omega\times\dot r\hat r$$

w/ $\alpha = \ddot\theta\hat k$, $\vec\omega = \dot\theta\hat k = 0$ (released from rest)

$$\vec a_g = \ddot\theta\hat k\times\left(-r\hat\imath - \frac{4}{3\pi}r\hat\jmath\right)$$

$$\vec a_g = \underbrace{+\frac{4}{3\pi}r\ddot\theta\hat\imath}_{a_{gx}=} \; \underbrace{-\ddot\theta r\hat\jmath}_{a_{gy}=} \quad \text{2 more eq's}$$

5) Dynamics:

$$\Sigma F_x: \; F_{Ax} = m a_{gx} = m\frac{4}{3}\frac{r}{\pi}\ddot\theta$$

$$\Sigma F_y: \; F_{Ay} = m(a_{gy} - g) = m(-r\ddot\theta + g)$$

$$\Sigma M_g: \; -\vec r\times\vec F = I\ddot\theta\hat k = m\frac{1}{4}\pi r^2\ddot\theta\hat k$$

$$\left(+r\hat\imath + \frac{4}{3}\frac{r}{\pi}\hat\jmath\right)\times\left(F_{Ax}\hat\imath + F_{Ay}\hat\jmath\right)$$

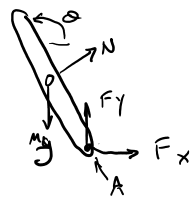
$$rF_{Ax} - \frac{4}{3}\frac{r}{\pi}F_{Ay} = \frac{1}{4}\pi r^2\ddot\theta$$

$$\begin{bmatrix} 1 & 0 & -m\frac{4r}{3\pi} \\ 0 & 1 & mr \\ r & -\frac{4}{3}\frac{r}{\pi}F_{Ay} & -\frac{m}{4}\pi r^2 \end{bmatrix}\begin{pmatrix} F_{Ax} \\ F_{Ay} \\ \ddot\theta \end{pmatrix} = \begin{pmatrix} 0 \\ +mg \\ 0 \end{pmatrix}$$

invert & solve.

The old truck has a ladder leaned against it, touching cab at the center of mass and at an angle of 120 degrees relative to horizontal. Find the acceleration of the truck when the ladder just begins to tip backward (CCW).

solution: 1) FBD:



2) list uk's: $F_x, F_y, N, a_{Ax}, a_{gx}, a_{gy}, \ddot\theta$

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3) list eq's: 3 - EOM.

ladder on verge of tipping: $N = 0$, $\ddot\theta = 0$ (3)

$\ddot{\theta} = 0$

4) Kinematics:

2 more from kinematics.

$$\vec{a}_g = \vec{a}_A + \frac{d^2}{dt^2}(\vec{r}) \quad w/\ \vec{r} = l/_2\, e^{i\theta}$$

$$\vec{a}_g = (a_{Ax}\hat{i} + 0\hat{j}) + \cancel{\ddot{l}/_2}\, e^{i\theta} + \cancel{\ddot{\theta}}\, l/_2\, i e^{i\theta} - \cancel{\dot{\theta}^2}\frac{l}{2} e^{i\theta} + 2\cancel{\dot{\theta}}\cancel{\dot{l}/_2}\, i e^{i\theta}$$

$$\vec{a}_g = a_{Ax}\hat{i} + 0\hat{j}$$

5) Dynamics:

ΣF_x: $F_x + \cancel{N}\, c(\theta - 90) = m\, a_{Ax}$

ΣF_y: $F_y + \cancel{N}\, s(\theta - 90) = m\, g$

ΣM_g: $-l/_2\, e^{i\theta} \times (F_x\hat{i} + F_y\hat{j}) = \frac{1}{12} m l^2 \cancel{\ddot{\theta}}$

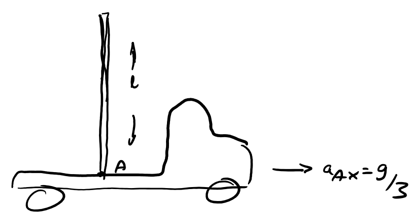
$$-l/_2(c\theta\hat{i} + s\theta\hat{j}) \times (F_x\hat{i} + F_y\hat{j}) = 0$$

$$-l/_2\, c\theta\, F_y - l/_2\, s\theta\, F_x = 0$$

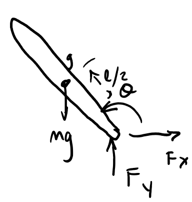
$$\begin{bmatrix} 1 & 0 & -m \\ 0 & 1 & 0 \\ -l/_2\, s\theta & -l/_2\, c\theta & 0 \end{bmatrix} \begin{pmatrix} F_{Ax} \\ F_{Ay} \\ a_{Ax} \end{pmatrix} = \begin{pmatrix} 0 \\ mg \\ 0 \end{pmatrix}$$

Solve a_{Ax}

The old truck has a flagpole that no one remembered to tie down. The truck starts moving (imagine truck goes in other direction) with constant acceleration of g/3, the flagpole is vertical and released from rest. What is the angular velocity of the flagpole when it hits the truck bed.



Solution: 1) FBD:



2) uk's: $F_x, F_y, \dot{\theta}, \ddot{\theta}, a_{gx}, a_{gy}$

(6)

3) eq's: 3, EOM

2 - kinematics,

1 - kinematic diff'l eq'n for $\dot{\theta}$

4) kinematics $\vec{a}_g = (a_{Ax}\hat{i} + 0\hat{j}) + r\ddot{\theta}\, i e^{i\theta}$ (note, ignoring $\dot{\theta}$ term for now)

$$a_{gx} = a_{Ax} - r\ddot{\theta}\, s\theta$$

$$a_{gy} = r\ddot{\theta}\, c\theta$$

5) EOM:

$$F_x = m(a_{Ax} - r\ddot{\theta}\, s\theta)$$

$$F_y - mg = m r \ddot{\theta}\, c\theta$$

$$-\tfrac{l}{2} e^{i\theta} \times (F_x \hat{i} + F_y \hat{j}) = \tfrac{1}{12} m l^2 \ddot{\theta}$$

$$-\tfrac{l}{2} (c\theta\, \hat{i} + s\theta\, \hat{j}) \times (F_x \hat{i} + F_y \hat{j}) = \tfrac{1}{12} m l^2 \ddot{\theta}$$

$$-\tfrac{l}{2} (c\theta\, F_y - s\theta\, F_x) = \tfrac{1}{12} m l^2 \ddot{\theta}$$

6)

$$\begin{bmatrix} 1 & 0 & m r\, s\theta \\ 0 & 1 & -m r\, c\theta \\ +\tfrac{l}{2} s\theta & -\tfrac{l}{2} c\theta & -\tfrac{m l^2}{12} \end{bmatrix} \begin{pmatrix} F_x \\ F_y \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} m\, a_{Ax} \\ m g \\ 0 \end{pmatrix}$$

Solve $\ddot{\theta} = f(\theta)$

7) more Kinematics:

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta} \frac{d\theta}{dt}$$

$$\ddot{\theta}\, d\theta = \dot{\theta}\, d\dot{\theta} = \tfrac{1}{2} \dot{\theta}^2 + C \quad \leftarrow \int \dot{\theta}\, d\dot{\theta} \quad \leftarrow c = \tfrac{1}{2} \dot{\theta}_0^2 \approx 0$$

$$\dot{\theta}^2 = 2 \ddot{\theta}\, d\theta$$

$$\dot{\theta} = \left[2 \int_{\theta=90}^{\theta=180} f(\theta) \right]^{1/2}$$