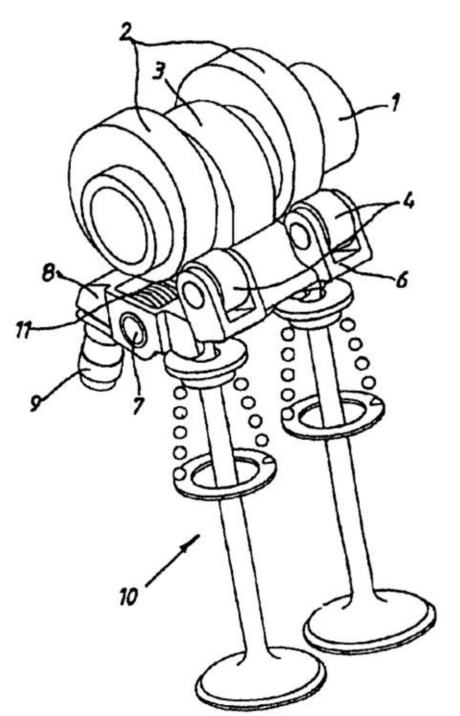
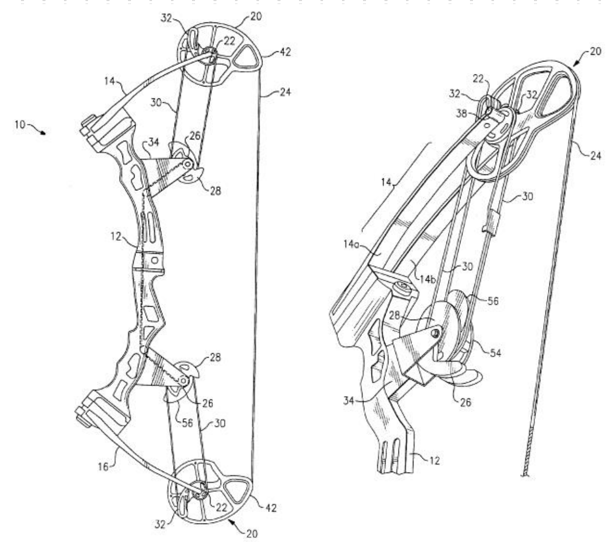
Part X: Cam System Design Via Conjugate Geometry

This section will review synthesis of basic cam systems and provide a set of analytical tools based on conjugate geometry. The areas covered in this section are:

1) Fundamentals of Cam Systems
    a. Types of Cams, Terminology
    b. Design Parameters
2) Cam Function Synthesis
    a. "law of Cams"
    b. Historical Methods (Kloomak and Muffley Curves)
    c. Modern Methods
3) Cam Profile Synthesis
    a. Conjugate Geometry and Loop Closure
    b. Disc Cams
    c. Wrapping Cams
4) Applications

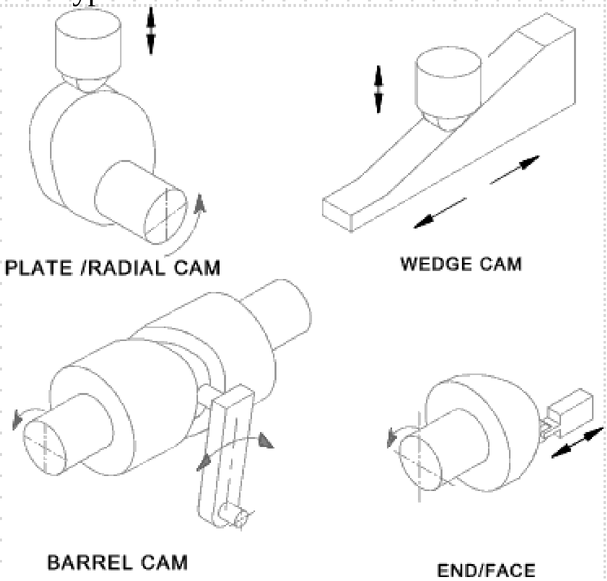
1) Fundamentals of Cam Systems
    a. Types of Cams,



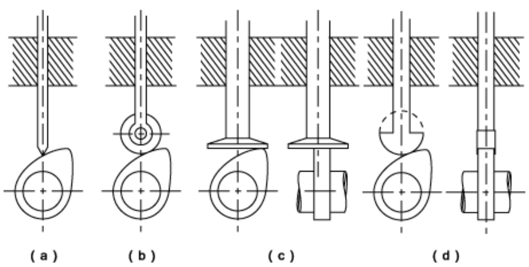


a. Terminology

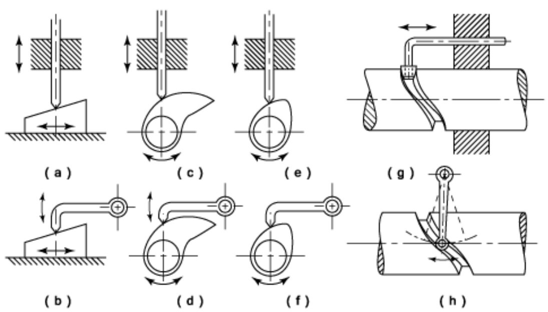
Types of cams:



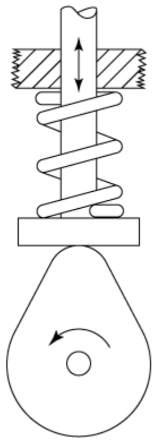
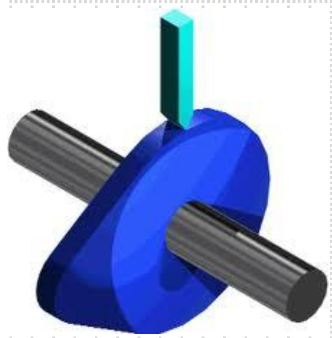
Type of follower:



Follower Form:



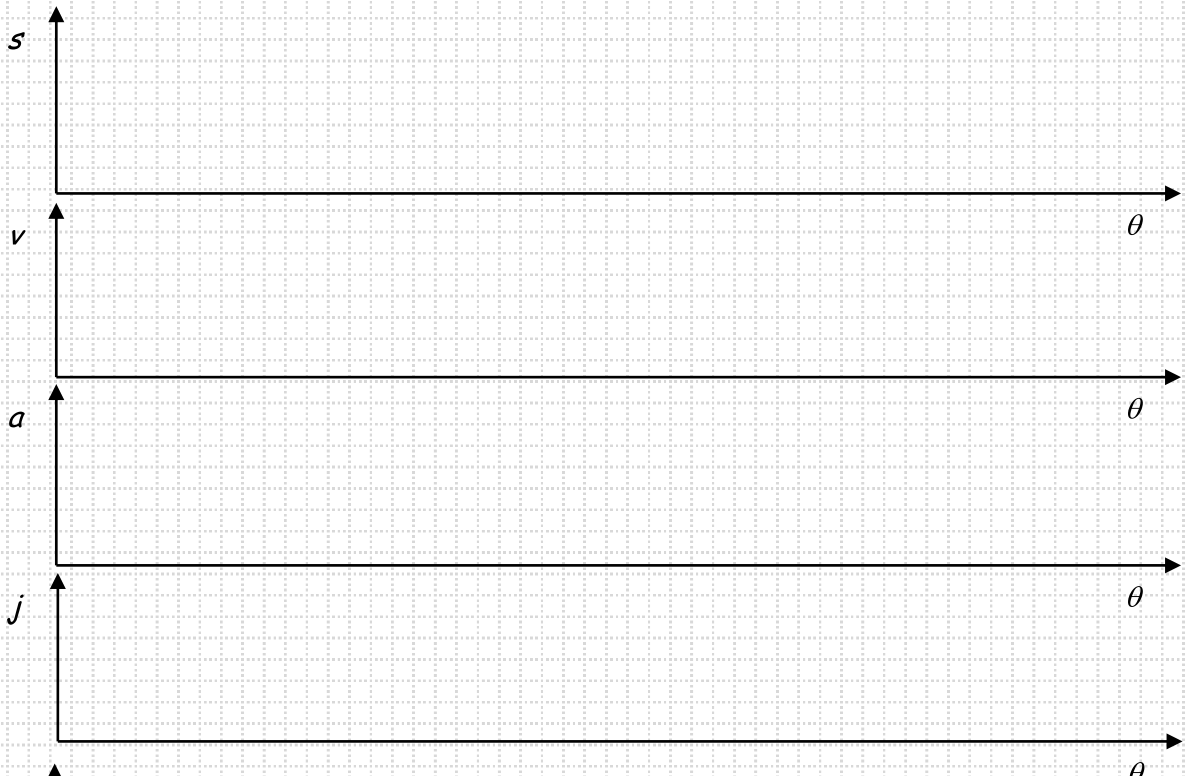
Practice naming cam systems

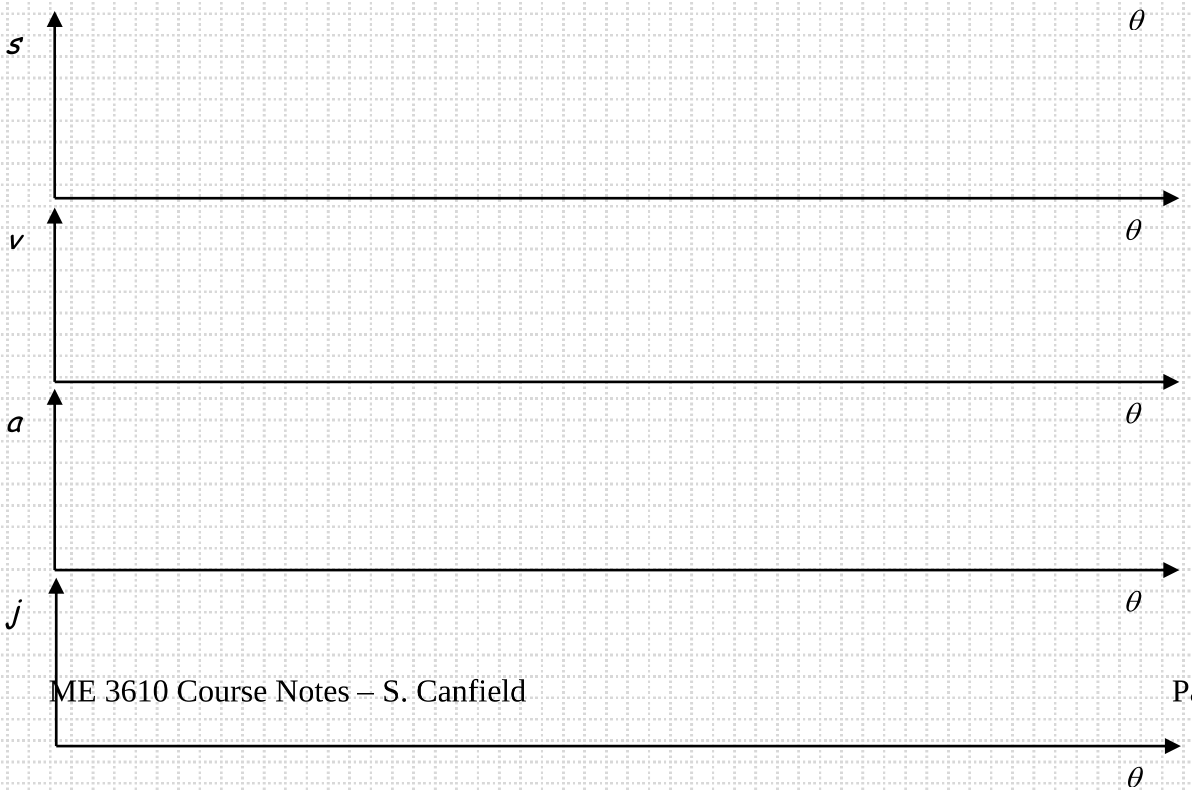



Design Parameters

2) Cam Function Synthesis
   a. “law of Cams”

   b. Sample Cam functions





c. General approaches: Historical Methods (Kloomak and Muffley Curves)

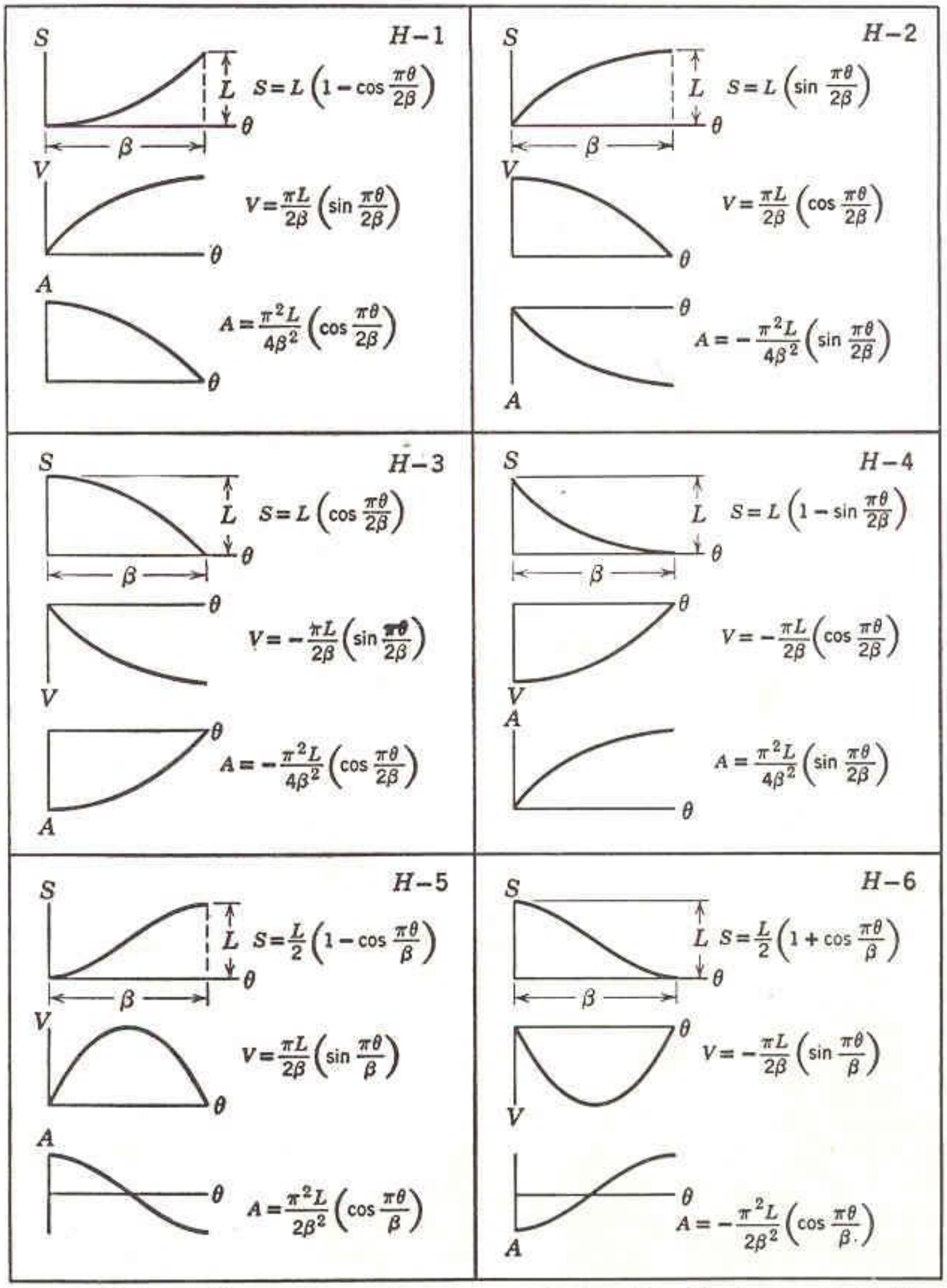


FIGURE 3.16 Harmonic motion characteristics: S = displacement, inches; V = velocity, inches per degree; A = acceleration, inches per degree squared. (M. Kloomok and R. V. Muffley, "Plate Cam Design—with Emphasis on Dynamic Effects," *Prod. Eng.*, February 1955.) *N. B.* For SI units, S = displacement, millimeters; V = velocity, millimeters per degree; A = acceleration, millimeters per degree squared.

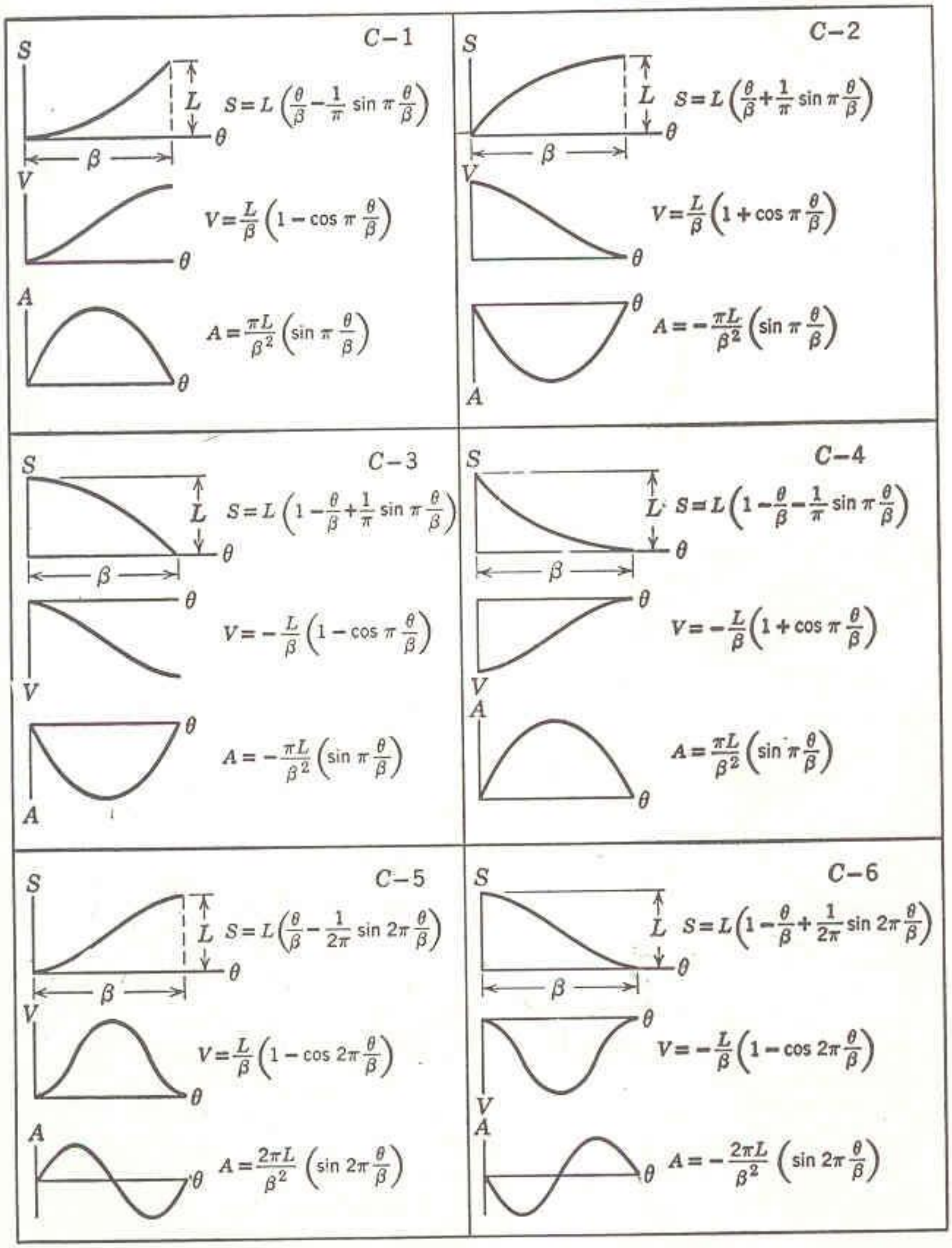
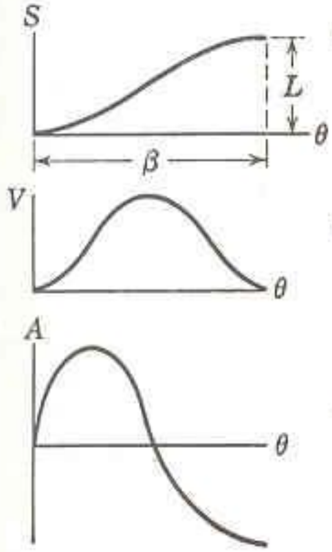


FIGURE 3.15 Cycloidal motion characteristics: S = displacement, inches; V = velocity, inches per degree; A = acceleration, inches per degree squared. (M. Kloomok and R. V. Muffley, "Plate Cam Design—with Emphasis on Dynamic Effects," *Prod. Eng.*, February 1955.) *N. B.* For SI units, S = displacement, millimeters; V = velocity, millimeters per degree; A = acceleration, millimeters per degree squared.

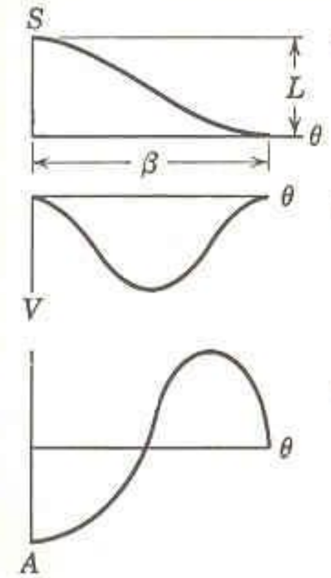


$$S = L\left[6.09755\left(\frac{\theta}{\beta}\right)^3 - 20.78040\left(\frac{\theta}{\beta}\right)^5 + 26.73155\left(\frac{\theta}{\beta}\right)^6 - 13.60965\left(\frac{\theta}{\beta}\right)^7 + 2.56095\left(\frac{\theta}{\beta}\right)^8\right]$$

$$V = \frac{L}{\beta}\left[18.29265\left(\frac{\theta}{\beta}\right)^2 - 103.90200\left(\frac{\theta}{\beta}\right)^4 + 160.38930\left(\frac{\theta}{\beta}\right)^5 - 95.26755\left(\frac{\theta}{\beta}\right)^6 + 20.48760\left(\frac{\theta}{\beta}\right)^7\right]$$

$$A = \frac{L}{\beta^2}\left[36.58530\left(\frac{\theta}{\beta}\right) - 415.60800\left(\frac{\theta}{\beta}\right)^3 + 801.94650\left(\frac{\theta}{\beta}\right)^4 - 571.60530\left(\frac{\theta}{\beta}\right)^5 + 143.41320\left(\frac{\theta}{\beta}\right)^6\right]$$

$P-1$



$$S = L\left[1.00000 - 2.63415\left(\frac{\theta}{\beta}\right)^2 + 2.78055\left(\frac{\theta}{\beta}\right)^5 + 3.17060\left(\frac{\theta}{\beta}\right)^6 - 6.87795\left(\frac{\theta}{\beta}\right)^7 + 2.56095\left(\frac{\theta}{\beta}\right)^8\right]$$

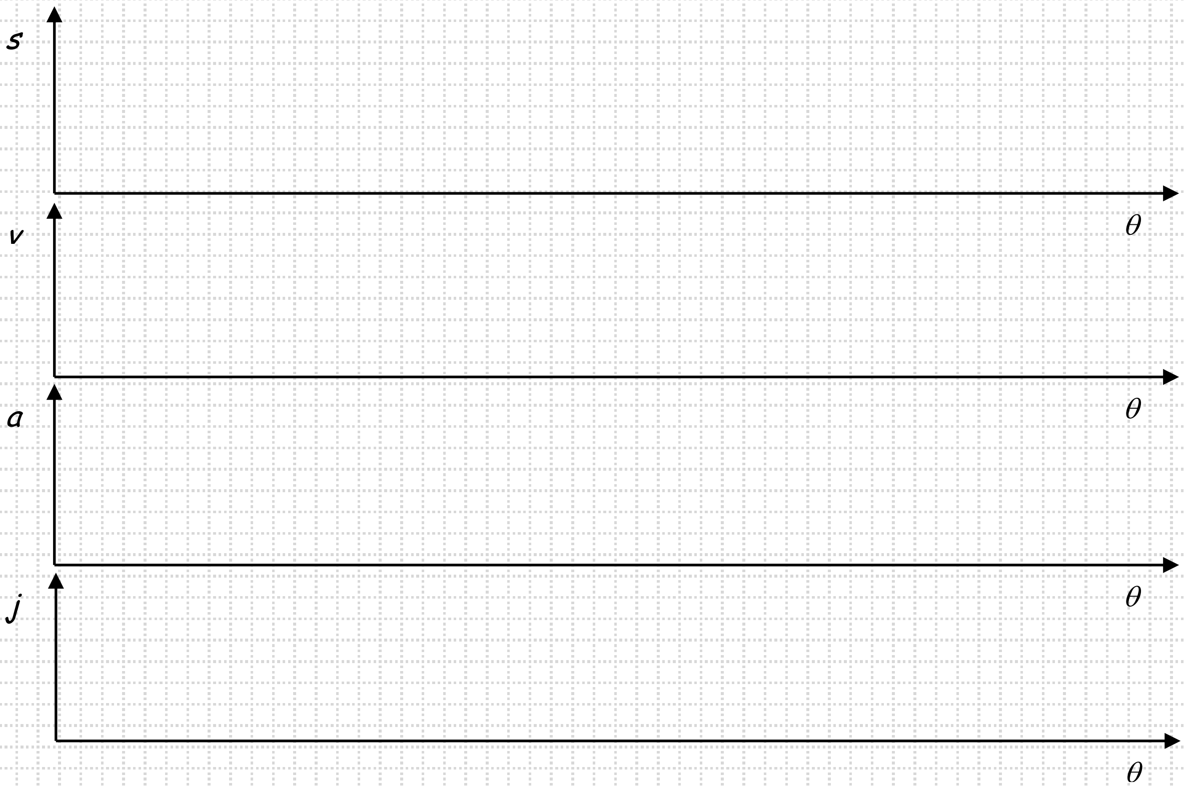
$$V = \frac{L}{\beta}\left[-5.26830\left(\frac{\theta}{\beta}\right) + 13.90275\left(\frac{\theta}{\beta}\right)^4 + 19.02360\left(\frac{\theta}{\beta}\right)^5 - 48.14565\left(\frac{\theta}{\beta}\right)^6 + 20.48760\left(\frac{\theta}{\beta}\right)^7\right]$$

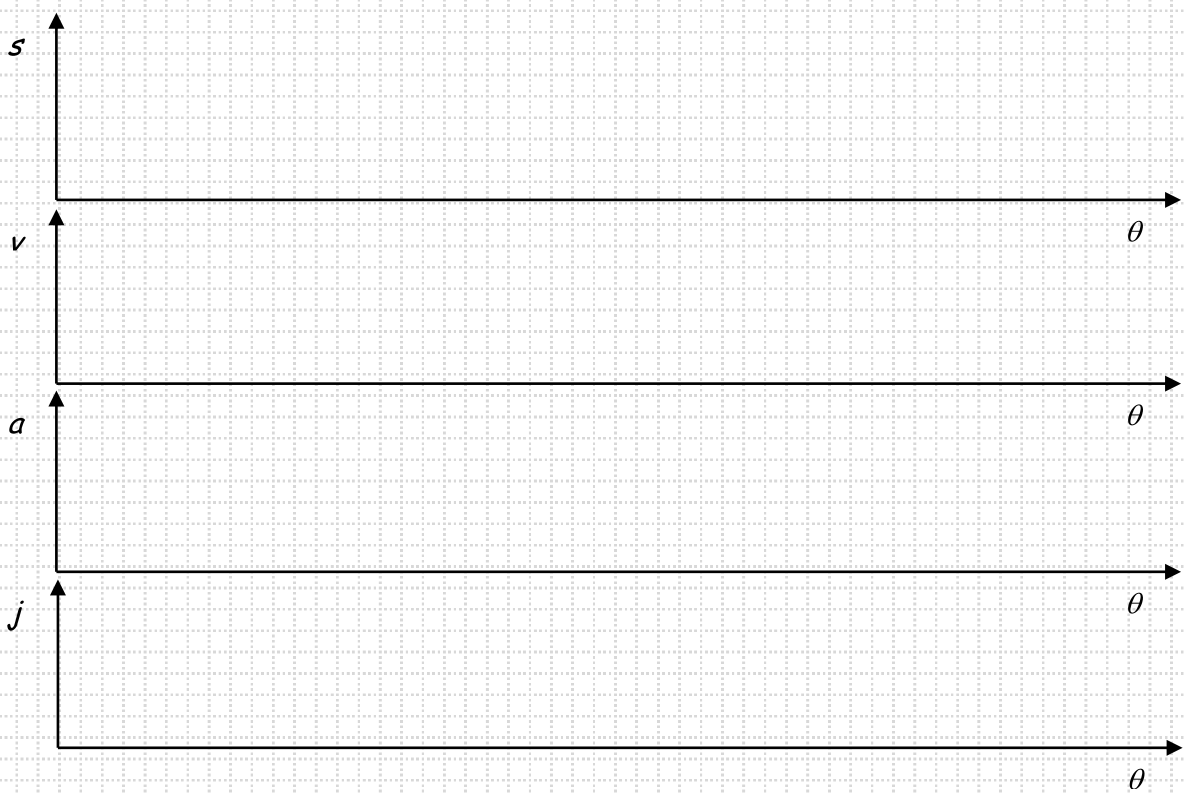
$$A = \frac{L}{\beta^2}\left[-5.26830 + 55.61100\left(\frac{\theta}{\beta}\right)^3 + 95.11800\left(\frac{\theta}{\beta}\right)^4 - 288.87390\left(\frac{\theta}{\beta}\right)^5 + 143.41320\left(\frac{\theta}{\beta}\right)^6\right]$$

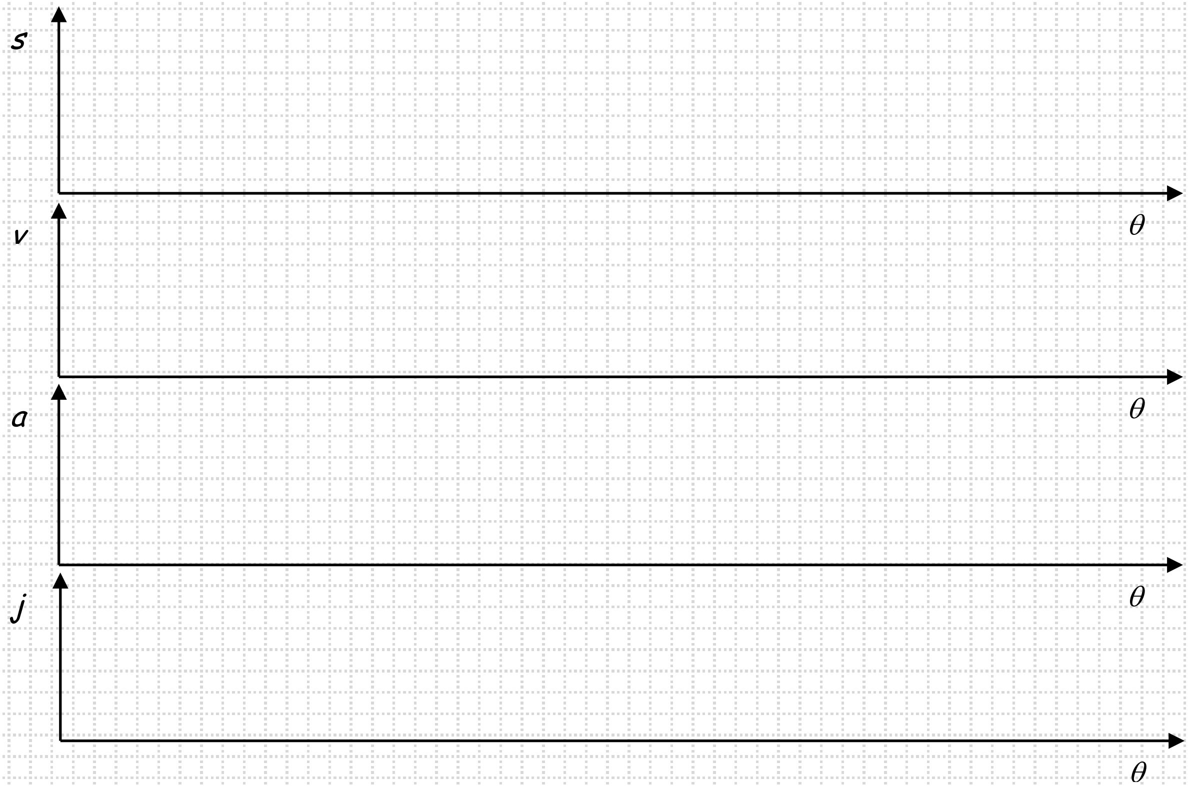
$P-2$

FIGURE 3.17 Eighth-power polynomial motion characteristics: S = displacement, inches; V = velocity, inches per degree; A = acceleration, inches per degree squared. (M. Kloomok and R. V. Muffley, "Plate Cam Design—with Emphasis on Dynamic Effects," *Prod. Eng.*, February 1955.) *N. B.* For SI units, S = displacements, millimeters; V = velocity, millimeters per degree; A = acceleration, millimeters per degree squared.

d. Modern Methods



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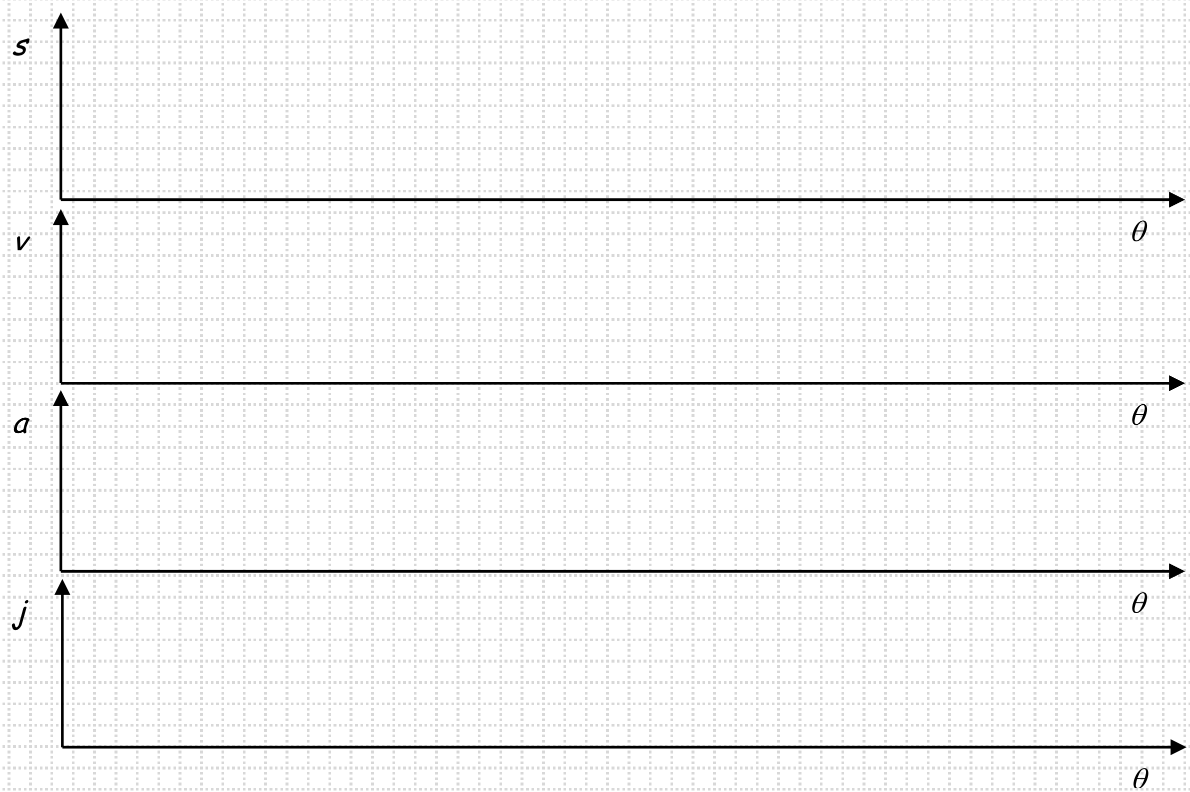
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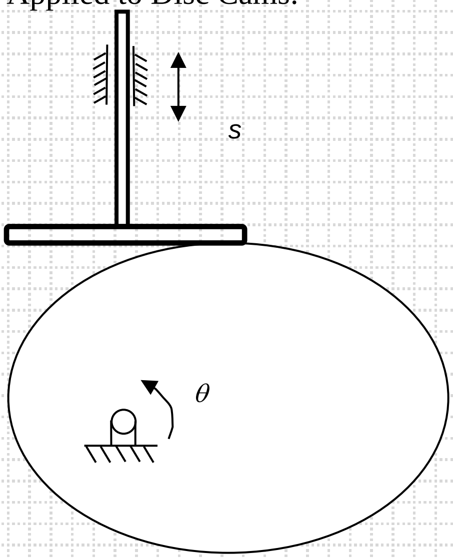
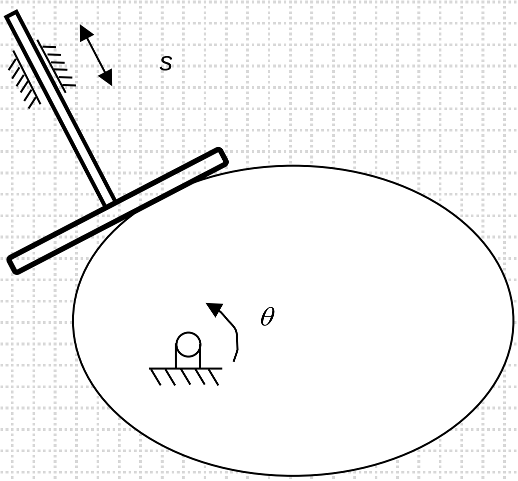
Cam Profile Synthesis
1: Conjugate Geometry and Loop Closure

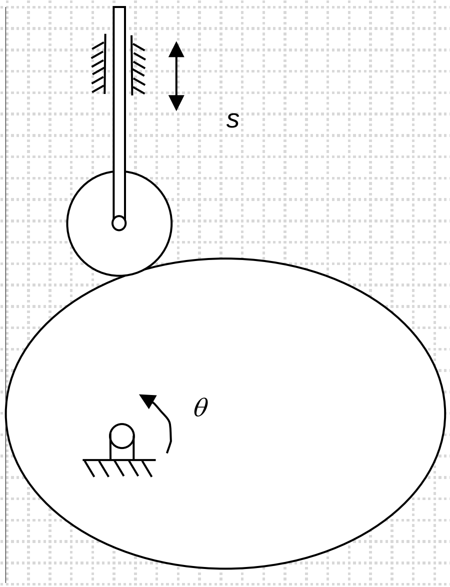
Law of Conjugate Geometry:

Loop Closure:

Applied to Disc Cams:



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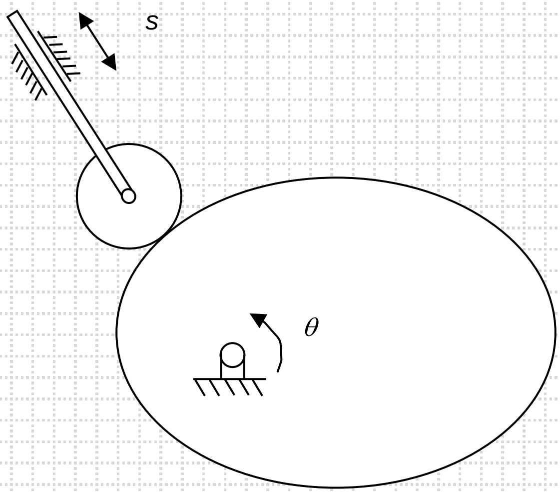
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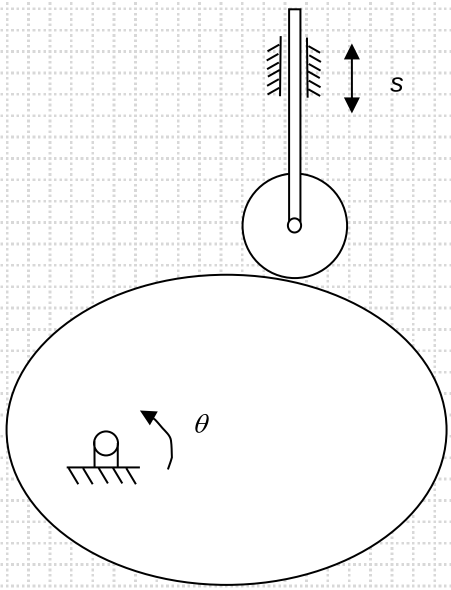
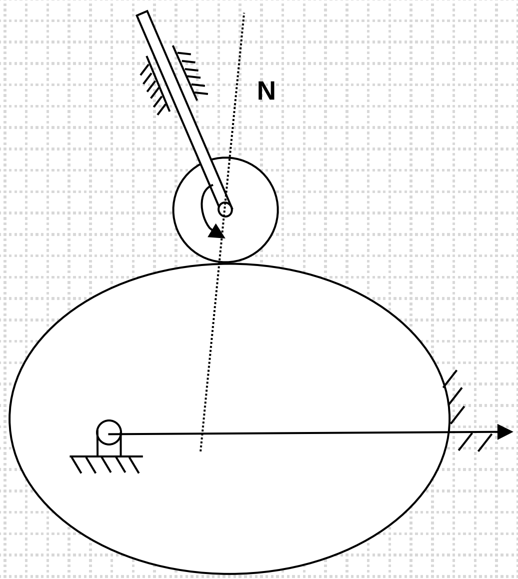




$\vec{P} = de^{i\theta} + sie^{i(\theta)} + rie^{i(\theta+\varphi)}$

$d\vec{P}/d\theta \cdot \hat{N} = 0$ Equations

$\vec{P}, \varphi$ Unknowns

To solve:

$d\vec{P}/d\theta = die^{i\theta} + s'ie^{i(\theta)} - se^{i(\theta)} - r(1+\varphi')e^{i(\theta+\varphi)}$

$\hat{N} = e^{i(\theta+\varphi+\pi/2)}$

$d\vec{P}/d\theta \cdot \hat{N} = \left[die^{i\theta} + s'ie^{i(\theta)} - se^{i(\theta)} - r(1+\varphi')e^{i(\theta+\varphi)}\right] \cdot e^{i(\theta+\varphi+\pi/2)} = 0$

$(d+s')\cos(-\varphi) - s\cos(-\varphi-90) - r(1+\varphi')\cos(90) = 0$

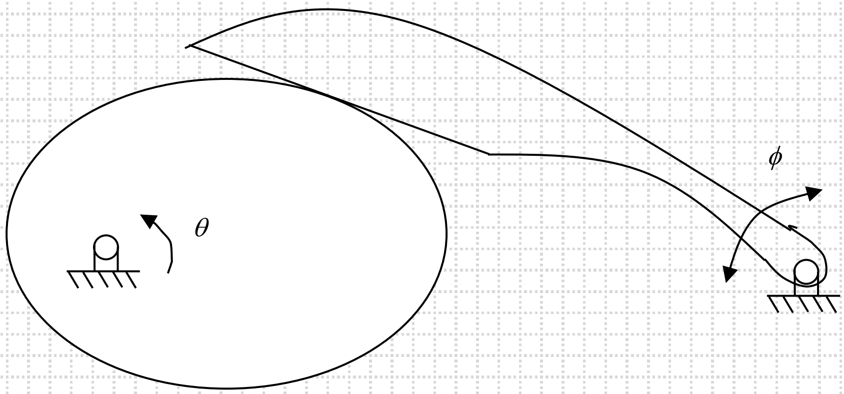
simplify,

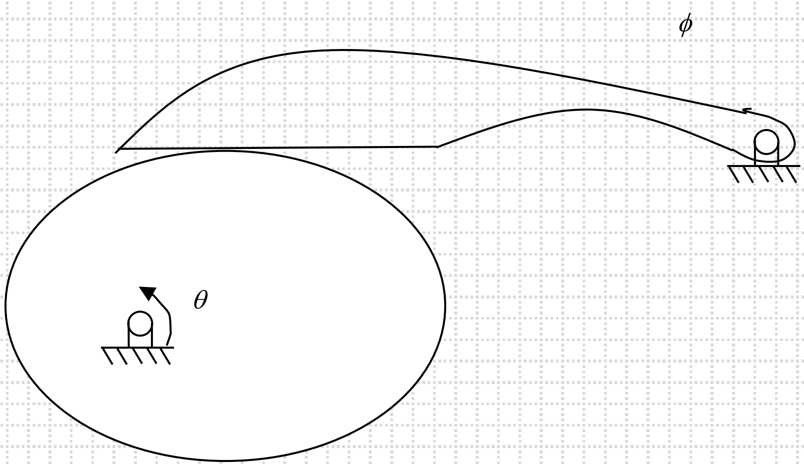
$(d+s')\cos(\varphi) + s\sin(\varphi) = 0$

$\varphi = \tan\left(\frac{-(d+s')}{s}\right)$

and

$\vec{P} = de^{i\theta} + sie^{i(\theta)} + rie^{i(\theta+\varphi)}$





$\vec{P} = Ce^{i\theta} + le^{i(\theta+\phi)}$

$d\vec{P}/d\theta \cdot \hat{N} = 0$ Equations

$\vec{P}, l$ Unknowns

To solve:

$$d\vec{P}/d\theta = Cie^{i\theta} + l'e^{i(\theta+\phi)} + li(1+\phi')e^{i(\theta+\phi)}$$

$$\hat{N} = e^{i(\theta+\phi+\pi/2)}$$

$$d\vec{P}/d\theta \cdot \hat{N} = \left[Cie^{i\theta} + l'e^{i(\theta+\phi)} + li(1+\phi')e^{i(\theta+\phi)}\right] \cdot e^{i(\theta+\phi+\pi/2)} = 0$$

$$C\cos(-\phi) + l'\cos(-90) + l(1+\phi')\cos(0) = 0$$

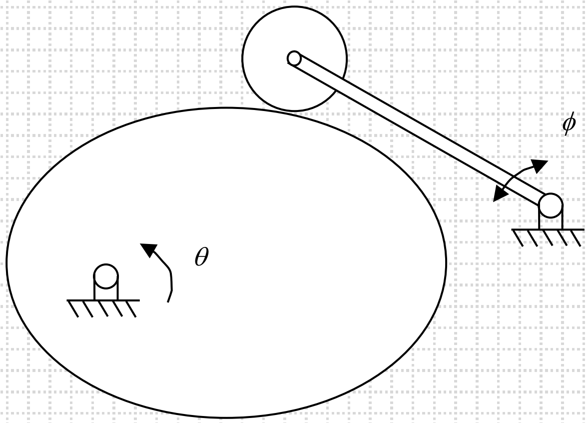
simplify,

$$C\cos(\phi)+l(1+\phi')=0$$

$$l=\left(\frac{-C\cos(\phi)}{(1+\phi')}\right)$$

and

$$\vec{P}=Ce^{i\theta}+\left(\frac{-C\cos(\phi)}{(1+\phi')}\right)e^{i(\theta+\phi)}$$



$\vec{P} = Ce^{i\theta} + le^{i(\theta+\phi)} + re^{i(\theta+\phi+\psi)}$

$d\vec{P}/d\theta \cdot \hat{N} = 0$ Equations

$\vec{P}, \psi$ Unknowns

To solve:

$d\vec{P}/d\theta = Cie^{i\theta} + li(1+\phi')e^{i(\theta+\phi)} + ri(1+\phi'+\psi')e^{i(\theta+\phi+\psi)}$

$\hat{N} = e^{i(\theta+\phi+\psi)}$

$d\vec{P}/d\theta \cdot \hat{N} = \left[Cie^{i\theta} + li(1+\phi')e^{i(\theta+\phi)} + ri(1+\phi'+\psi')e^{i(\theta+\phi+\psi)}\right] \cdot e^{i(\theta+\phi+\psi)} = 0$

$C\cos(90-\phi-\psi) + l(1+\phi')\cos(90-\psi) + r(1+\phi'+\psi')\cos(90) = 0$

simplify,

$C\sin(\phi+\psi) + l(1+\phi')\sin(\psi) = 0$

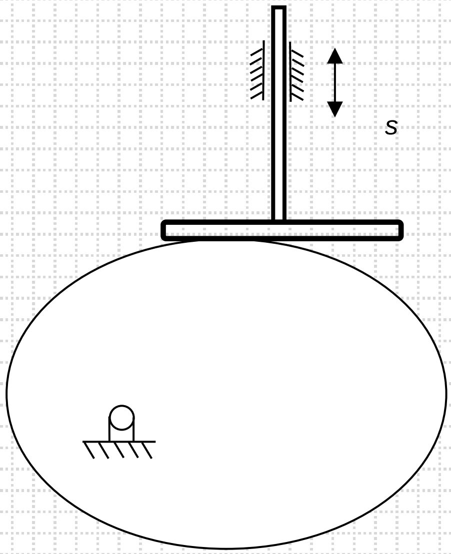
$C[\sin(\phi)\cos(\psi) + \cos(\phi)\sin(\psi)] + l(1+\phi')\sin(\psi) = 0$

$C\sin(\phi)\cos(\psi) + [C\cos(\phi) + l(1+\phi')]\sin(\psi) = 0$

$$\tan(\psi) = \frac{-C\sin(\phi)}{C\cos(\phi) + l(1+\phi')}$$

and

$\vec{P} = Ce^{i\theta} + le^{i(\theta+\phi)} + re^{i(\theta+\phi+\psi)}$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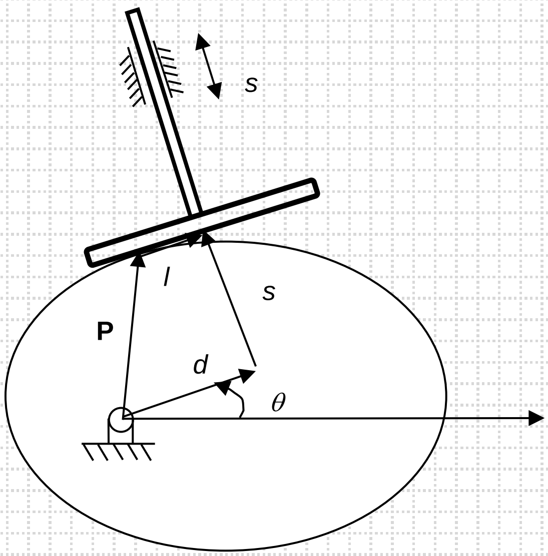

Wrapping Cams:

$$\vec{P} = de^{i\theta} + sie^{i(\theta)} - le^{i(\theta)}$$

$$d\vec{P}/d\theta \cdot \widehat{N} = 0$$

Equations

$\vec{P}, l$ Unknowns

To solve:

$$d\vec{P}/d\theta = die^{i\theta} + s'ie^{i(\theta)} - se^{i(\theta)} - l'e^{i(\theta)} - lie^{i(\theta)}$$

$$\hat{N} = e^{i(\theta + \pi/2)}$$

$$d\vec{P}/d\theta \cdot \hat{N} = \left[die^{i\theta} + s'ie^{i(\theta)} - se^{i(\theta)} - l'e^{i(\theta)} - lie^{i(\theta)}\right] \cdot e^{i(\theta + \pi/2)} = 0$$

$$(d + s' + l)\cos(0) - (s + l')\cos(90) = 0$$

simplify,

$$d + s' + l = 0$$

$$l = -(d + s')$$

and

$$\vec{P} = de^{i\theta} + sie^{i(\theta)} + (d + s')e^{i(\theta)}$$

Examples: Cam design
http://www.circletrack.com/techarticles/ctrp_0701_camshaft_design_science/viewall.html
http://www.tildentechnologies.com/Cams/index.html
http://en.wikipedia.org/wiki/Camshaft
http://www.rbracing-rsr.com/camshaft.html
http://www.enginebasics.com/Engine%20Basics%20Root%20Folder/Basic%20Camshaft%20Understanding.html

http://www.compcams.com/Company/CC/cam-specs/

Assignment:
Design a two cams based on the following specs:
http://www.compcams.com/Company/CC/cam-specs/Details.aspx?csid=790&sb=0

One cam should employ a flat-faced follower, second a roller follower.

Complete the following:

1) Plot of SVAJ for the follower
2) Plot the two cams in matlab.
3) Overlay to highlight the differences.
4) (for fun only) animate the cam/follower