Part VIII: Acceleration Analysis of Mechanisms

This section will review the most common and currently practiced methods for completing the kinematics analysis of mechanisms; describing motion through velocity and acceleration. This section of notes will be divided among the following topics:

1) Acceleration analysis: analytical techniques
2) Acceleration analysis, Classical techniques

1) Overview of acceleration analysis of mechanisms:

Important features associated with velocity and acceleration analysis:

a. Kinematics:

b. Types of equations that result

c. General approach/strategy

d. Uses/Applications

2) Acceleration Analysis: Analytical Techniques:

As in velocity analysis, the acceleration analysis of a mechanism is performed by taking the derivative of the position equations w.r.t. time. When acceleration analysis is performed, it is of particular interest to note the frame in which derivatives are taken, since Newton's second equation requires acceleration with respect to an inertial frame (also called Newtonian frame). In our analytical acceleration analysis of mechanisms, we will ensure inertial accelerations by having our mechanism grounded to an appropriate inertial frame.

The second derivative of the position vector equations will yield vector equations in acceleration (second derivatives of constraint equations will yield scalar constraints in acceleration). The unknown accelerations are linear and are solved using linear algebra. The approach proceeds as follows:

Step 1: Get the Position solution

Step 2: Get the velocity Solution

Step 3: list the acceleration unknowns

> Unknowns in the model, unknown accelerations of points of interest

Step 4: Get the equations:

Take second derivative of a loop/constraint equations from the model
Second derivative of a chain going from ground out to a point of interest

Step 5: Solve accelerations as a linear system of equations

1) Isolate unknowns on one side of equation knowns on the other
2) Expand into scalar equations:
3) Cast into matrix form
4) Solve in excel, matlab or your calculator.

3) Review the second derivative of a vector.
Review: Taking time derivatives of a vector.

In cartesian vector notation:

$$\bar{r} = r\hat{r} = r\hat{r}$$
$$a = \frac{d^2}{dt^2}\bar{r}$$

In complex polar notation:

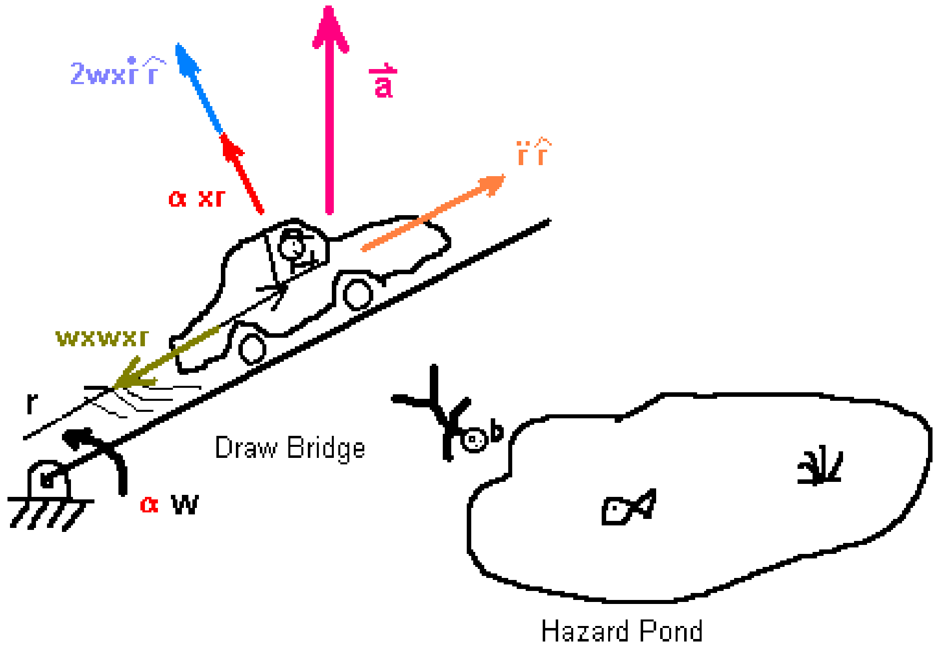
$$\bar{r} = re^{i\theta}$$
$$a = \frac{d^2}{dt^2}\bar{r}$$

Expand into Scalar Components

Summary:

Table: Summary of Vector Derivative and Scalar Components

Explanation of Acceleration Terms – or – The Dynamics of the Dukes:



For a loop equation:

$$r_2 + r_3 = r_1 + r_4$$

More discussion of acceleration: Examples of each term in the acceleration equation.

Linear acceleration term

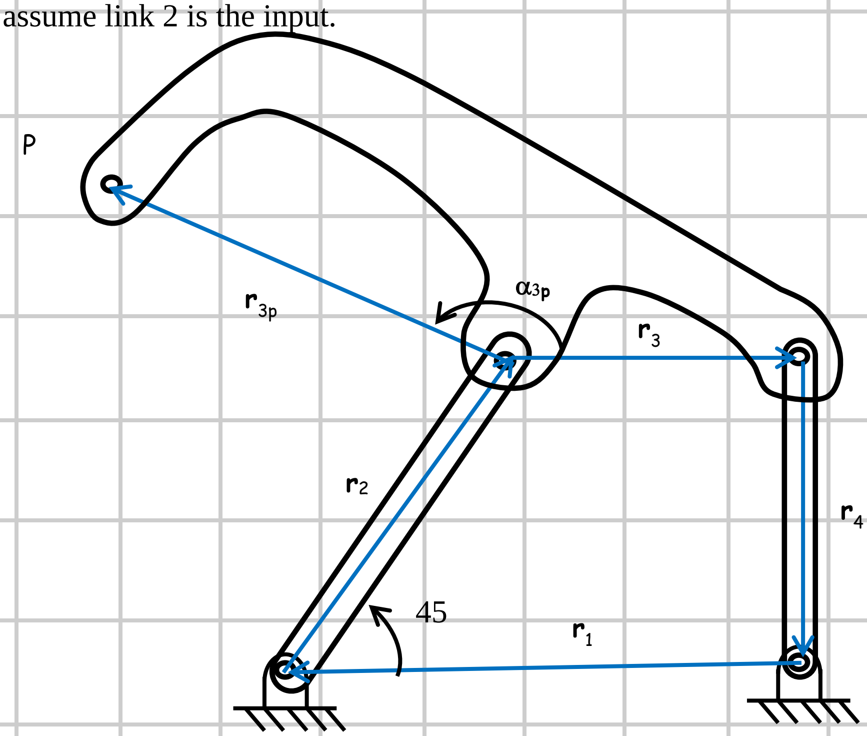
Angular acceleration term

Centripetal acceleration term

Coriolis acceleration term

Performing Acceleration Analysis – The Process:
Example 1: Bobcat 650S loader: http://www.youtube.com/watch?v=aqK0qsXH3e4
Find the acceleration unknowns in the model and the acceleration of the bucket pivot point (P)

First, consider an extremely simplified model that ignores the bucket, ignores the input cylinder, and assumes that links 1 and 3 are horizontal, link 4 is vertical and link 2 is at 45 degrees. Also, assume link 2 is the input.



Given r_1 = 161 cm; r_2 = 141 cm, r_3 = 120 cm, r_4 = 100 cm, θ_2 = 45deg, $\dot{\theta}_2$ = .5 rad/s, $\dot{\theta}_2$ = .5 rad/s2, r_{3p} = 200, α_{3p} = 135deg
From Velocity problem: $\dot{\theta}_3 = -.4154$ $\dot{\theta}_4 = .4985$ $\dot{\theta}_{3p} = -.4154$

Solution:

Step 1: get the position solution:

1. Schematic (see above)
2. Mobility: $M = 3(4-1) - 2(4) - 0 = 1$
3. Vector model (see above)
4. position unknowns $\theta_3, \theta_{3p}, \theta_4;\ \theta_2, = input$
5. Equations

$$L1: \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = 0$$
$$CE: \theta_{3p} = \theta_3 + \alpha_{3p}$$

Step 2: List acceleration unknowns:

Step 3: Identify and write equations:

2 equations from loop vector equation

1 equation from constraint equation

2 equations of a vector chain from ground to point P

Total = 5 equations

Step 4: Expand equations into scalar form:

Step 5: Solve:

Discussion:

The results indicate…. A few important notes:

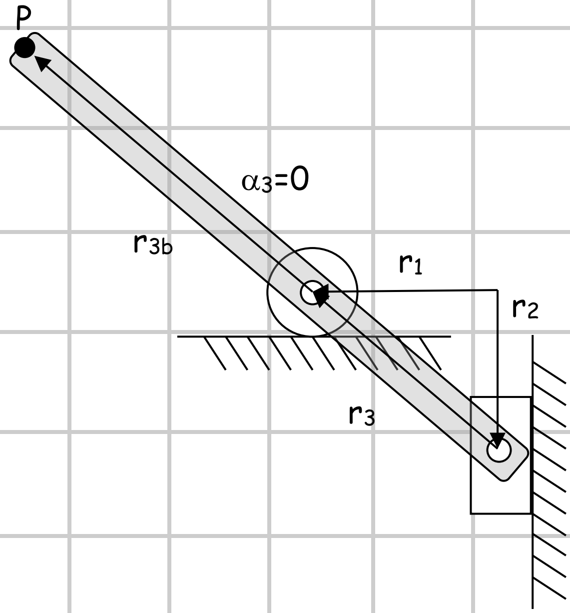
1) The solution is linear in the accelerations

2) This solution is valid only at the position shown in the figure above. As the loader moves, this solution is no longer valid, the angles need to be updated.

3) Even if the input is driven at a constant velocity (zero acceleration), the remaining mechanism can/will have acceleration terms.

Example #2:

Given the floating arm trebuchet shown as a schematic below. Assume r_2, r_2_dot r_2_dbldot are the inputs, r_1 = 173cm, r_2 = 100cm, theta3 = 150 deg., r_3 = 200cm, r_{3b} 300 cm, and r_2_dot = 250 cm/s and r_2_dot = 981 cm/s^2 solve for the unknown acceleration terms in the model equations and the acceleration of P, x and y coordinates.



Solution:

Step 1: get the position solution:

1. Schematic (see above)
2. Mobility: $M = 3(4-1) - 2(4) - 0 = 1$
3. Vector model (see above)
4. Position unknowns $r_1, \theta_3, \theta_{3b};\ r_2, = input$
5. Equations

$$L1: -\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = 0$$
$$CE: \theta_{3b} = \theta_3 + \alpha_3$$

Also, from the velocity solution:

$\dot{r}_1 = -144.3, \dot{\theta}_3 = -1.443,\ \dot{\theta}_{3b} = -1.443, V_{px} = 360.75, V_{py} = 374.84$

Step 2: List acceleraton unknowns:

$$\ddot{r}_1, \ddot{\theta}_3, \ddot{\theta}_{3b}, \mathbf{A}_{\boldsymbol{p}};\ \ddot{r}_2, = input$$

five unknowns - need five equations.

Step 3: Identify and write equations:

$$\frac{d^2}{dt^2}(L1): -\ddot{r}_1 e^{i\theta_1} + \ddot{r}_2 e^{i\theta_2} + r_3\ddot{\theta}_3 i e^{i\theta_3} - r_3\dot{\theta}_3^2 e^{i\theta_3} = 0$$

2 equations from loop vector equation

$$\frac{d^2}{dt^2}(CE): \ddot{\theta}_{3b} = \ddot{\theta}_3$$

1 equation from constraint equation

$$\mathbf{A}_{\boldsymbol{p}} = \frac{d^2}{dt^2}\left(\overrightarrow{OP}\right) = \frac{d^2}{dt^2}(\mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_{3b})$$
$$= \ddot{r}_2 e^{i\theta_2} + r_3\ddot{\theta}_3 i e^{i\theta_3} - r_3\dot{\theta}_3^2 e^{i\theta_3} + r_{3b}\ddot{\theta}_{3b} i e^{i\theta_{3b}} - r_{3b}\dot{\theta}_{3b}^2 e^{i\theta_{3b}}$$

2 equations of a vector chain from ground to point P

Total = 5 equations

Step 4: Expand equations into scalar form:

$$\frac{d^2}{dt^2}(L1): -\ddot{r}_1 e^{i\theta_1} + \ddot{r}_2 e^{i\theta_2} + r_3\ddot{\theta}_3 i e^{i\theta_3} - r_3\dot{\theta}_3^2 e^{i\theta_3} = 0$$
$$L1_x: -\ddot{r}_1 c_1 + \ddot{r}_2 c_2 - r_3\ddot{\theta}_3 s_3 - r_3\dot{\theta}_3{}^2 c_3 = 0$$
$$L1_y: -\ddot{r}_1 s_2 + \ddot{r}_2 s_2 + r_3\ddot{\theta}_3 c_3 - r_3\dot{\theta}_3{}^2 s_3 = 0$$
$$CE: \dot{\theta}_{3b} = \dot{\theta}_3$$
$$\mathbf{A}_{\boldsymbol{p}} = \ddot{r}_2 e^{i\theta_2} + r_3\ddot{\theta}_3 i e^{i\theta_3} - r_3\dot{\theta}_3^2 e^{i\theta_3} + r_{3b}\ddot{\theta}_{3b} i e^{i\theta_{3b}} - r_{3b}\dot{\theta}_{3b}^2 e^{i\theta_{3b}}$$
$$A_{px} = \ddot{r}_2 c_2 - r_3\ddot{\theta}_3 s_3 - r_3\dot{\theta}_3{}^2 c_3 - r_{3b}\ddot{\theta}_{3b} s_{3b} - r_{3b}\dot{\theta}_{3b}{}^2 c_{3b}$$
$$A_{py} = \ddot{r}_2 s_2 + r_3\ddot{\theta}_3 c_3 - r_3\dot{\theta}_3{}^2 s_3 + r_{3b}\ddot{\theta}_{3b} c_{3b} - r_{3b}\dot{\theta}_{3b}{}^2 s_{3b}$$

Step 5: Solve:

<u>Discussion:</u>

The results indicate …. Two important notes:

1) The solution is linear in the accelerations.

2) This solution is valid only at the position shown in the figure above. As the trebuchet moves, this solution is no longer valid, the angles need to be updated.