Part VI: Velocity and Acceleration Analysis of Mechanisms

This section will review the most common and currently practiced methods for completing the kinematics analysis of mechanisms; describing motion through velocity and acceleration. This section of notes will be divided among the following topics:

1) Overview of velocity and acceleration analysis of mechanisms
2) Velocity analysis: analytical techniques
3) Velocity analysis: Classical techniques (instant centers, centrodes, etc.)
4) Static force analysis, mechanical advantage
5) Acceleration analysis: analytical techniques
6) Acceleration analysis, Classical techniques

1) Overview of velocity and acceleration analysis of mechanisms:

Important features associated with velocity and acceleration analysis:

a. Kinematics:

b. Types of equations that result

c. General approach/strategy

d. Uses/Applications

2) Velocity Analysis: Analytical Techniques

The standard approach to velocity analysis of a mechanism is to take derivative of the position equations w.r.t. time. (Note, alternative approaches, such as those termed influence coefficients, can be performed by first taking the partial derivative with respect to an alternate parameter multiplied by the time derivative of that parameter)

The position equations that we consider are predominantly loop closure and constraint equations. The approach will take the derivatives of these equations, expand into scalar equations and solve for the unknowns (all problems will be linear in the unknowns!).

Process:

1) Take first derivative of a loop equation:

2) move knowns to one side of equation, unknowns to the other side

3) Expand into scalar equations:

4) Cast into matrix form and solve:

Review: Taking time derivatives of a vector.

In cartesian vector notation:

$$\bar{r} = r\hat{r} = r\hat{r}$$

$$\mathbf{v} = \frac{d}{dt}\bar{r}$$

In complex polar notation:

$$\bar{r} = re^{i\theta}$$

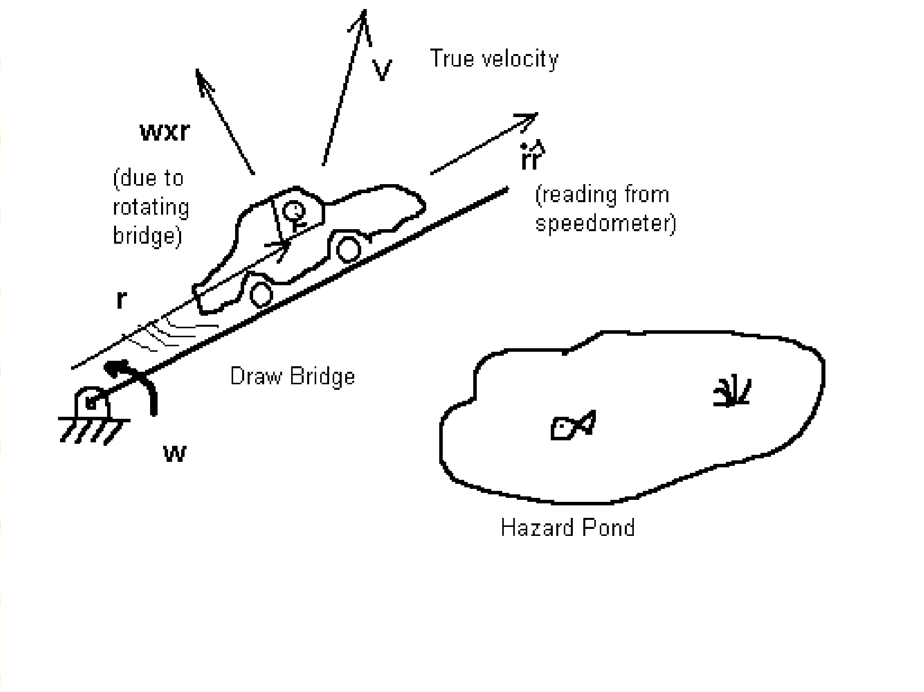
$$\mathbf{v} = \frac{d}{dt}\bar{r}$$

<u>Expand into scalar components:</u>

Summary:

Table: Summary of Vector Derivative and Scalar Components		

Explanation of Velocity Terms – or – The Dynamics of the Dukes:



For a loop equation:

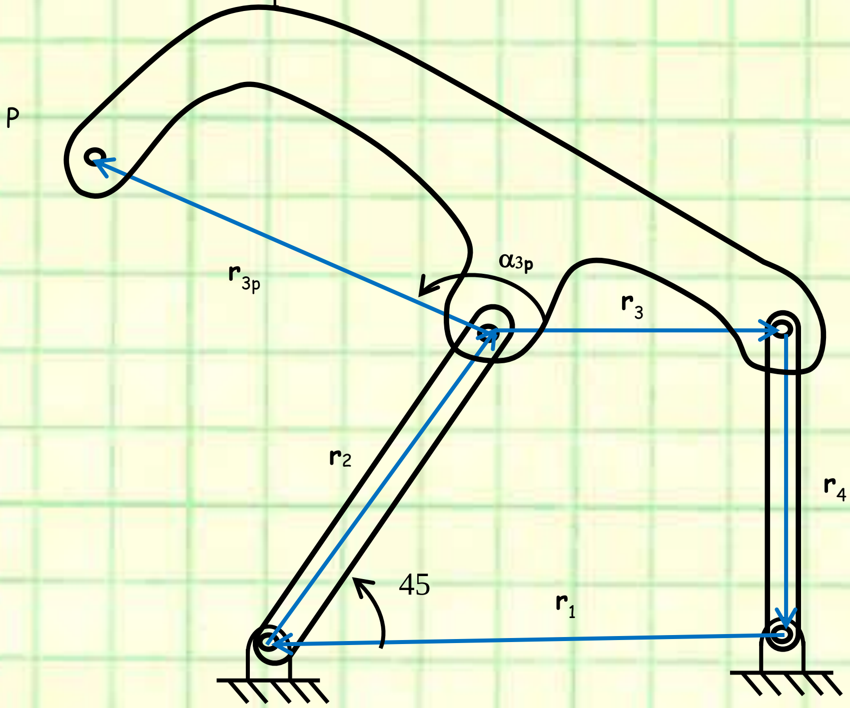
$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4$$

$$\dot{r}_2 e^{i\theta_2} + r_2 i\dot{\theta} e^{i\theta_2} + \dot{r}_3 e^{i\theta_3} + r_3 i\dot{\theta} e^{i\theta_3} = \dot{r}_1 e^{i\theta_1} + r_1 i\dot{\theta} e^{i\theta_1} + \dot{r}_4 e^{i\theta_4} + r_4 i\dot{\theta} e^{i\theta_4}$$

Performing Velocity Analysis – The Process:

Example 1: Bobcat 650S loader: http://www.youtube.com/watch?v=aqK0qsXH3e4

First, consider an extremely simplified model that ignores the bucket, ignores the input cylinder, and assumes that links 1 and 3 are horizontal, link 4 is vertical and link 2 is at 45 degrees. Also, assume link 2 is the input.



Given r_1 = 161 cm; r_2 = 141 cm, r_3 = 120 cm, r_4 = 100 cm, θ_2 = 45deg, $\dot{\theta}_2$ = .5 rad/s, r_{3p} = 200, α_{3p} = 135deg

Solution:

Step 1: get the position solution:

1. Schematic (see above)
2. Mobility: $M = 3(4-1) - 2(4) - 0 = 1$
3. Vector model (see above)
4. position unknowns $\theta_3, \theta_{3p}, \theta_4;\ \theta_2, = input$
5. Equations

$$L1: \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = 0$$
$$CE: \theta_{3p} = \theta_3 + \alpha_{3p}$$

Step 2: List velocity unknowns:

$$\dot{\theta}_3, \dot{\theta}_{3p}, \dot{\theta}_4, \mathbf{V}_{\boldsymbol{p}};\ \dot{\theta}_2, = input$$

five unknowns - need five equations.

Step 3: Identify and write equations: (note r_1 through r_4 are constant length and theta1 is constant angle so derivative is 0)

$$\frac{d}{dt}(L1): r_2\dot{\theta}_2 i e^{i\theta_2} + r_3\dot{\theta}_3 i e^{i\theta_3} + r_4\dot{\theta}_4 i e^{i\theta_4} = 0$$

2 equations from loop vector equation

$$\frac{d}{dt}(CE): \dot{\theta}_{3p} = \dot{\theta}_3$$

1 equation from constraint equation

$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}\left(\mathbf{r}_2 + \mathbf{r}_{3p}\right) = r_2\dot{\theta}_2 i e^{i\theta_2} + r_{3p}\dot{\theta}_{3p} i e^{i\theta_{3p}}$$

2 equations of a vector chain from ground to point P

Total = 5 equations

Step 4: Expand equations into scalar form:

$$\frac{d}{dt}(L1): r_2\dot{\theta}_2 i e^{i\theta_2} + r_3\dot{\theta}_3 i e^{i\theta_3} + r_4\dot{\theta}_4 i e^{i\theta_4} = 0$$
$$L1_x: -r_2\dot{\theta}_2 s_2 - r_3\dot{\theta}_3 s_3 - r_4\dot{\theta}_4 s_4 = 0$$
$$L1_y: +r_2\dot{\theta}_2 c_2 + r_3\dot{\theta}_3 c_3 + r_4\dot{\theta}_4 c_4 = 0$$
$$CE: \dot{\theta}_{3p} = \dot{\theta}_3$$
$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}\left(\mathbf{r}_2 + \mathbf{r}_{3p}\right) = r_2\dot{\theta}_2 i e^{i\theta_2} + r_{3p}\dot{\theta}_{3p} i e^{i\theta_{3p}}$$
$$V_{px} = -r_2\dot{\theta}_2 s_2 - r_{3p}\dot{\theta}_{3p} s_{3p}$$
$$V_{py} = r_2\dot{\theta}_2 c_2 + r_{3p}\dot{\theta}_{3p} c_{3p}$$

Step 5: Solve:

$$L1_x: -141 * 0.5 * \sin(45) - 120 * \dot{\theta}_3 * \sin(0) - 100 * \dot{\theta}_4 * \sin(-90) = 0$$
$$L1_y: +141 * 0.5 * \cos(45) + 120 * \dot{\theta}_3 * \cos(0) + 100 * \dot{\theta}_4 * \cos(-90) = 0$$

$L1_x: -49.85 - 0 + 100 * \dot{\theta}_4 = 0 \rightarrow \dot{\theta}_4 = .4985$

$L1_y: +49.85 + 120 * \dot{\theta}_3 + 0 = 0 \rightarrow \dot{\theta}_3 = -.4154$

$CE: \dot{\theta}_{3p} = \dot{\theta}_3 \rightarrow \dot{\theta}_{3p} = -.4154$

$$V_{px} = -141 * 0.5 * \sin(45) - 200 * -.4154 * \sin(135) = 8.896$$

$$V_{py} = +141 * 0.5 * \cos(45) + 200 * -.4154 * \cos(135) = 108.6$$

Discussion:

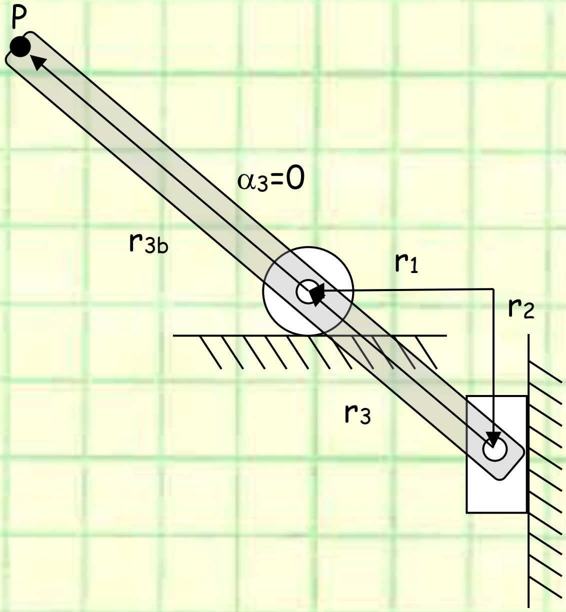
The results indicate that the loader pin P is moving up and slightly to the right. Two important notes:

1) The solution is linear in the velocities. This means that if the input velocity were doubled, the output velocity would double.

2) This solution is valid only at the position shown in the figure above. As the loader moves, this solution is no longer valid, the angles need to be updated. This is demonstrated in example 3.

Example #2:

Given the floating arm trebuchet shown as a schematic below. Assume r_2, r_2_dot are the inputs, r_1 = 173cm, r_2 = 100cm, theta3 = 150 deg., r_3 = 200cm, r_{3b} 300 cm, and r_2_dot = 250 cm/s solve for the unknown velocity terms in the model equations and the velocity of P, x and y coordinates.



Solution:

Step 1: get the position solution:

1. Schematic (see above)
2. Mobility: $M = 3(4-1) - 2(4) - 0 = 1$
3. Vector model (see above)
4. Position unknowns $r_1, \theta_3, \theta_{3b};\ r_2, = input$
5. Equations

$$L1:\ -\mathbf{r_1} + \mathbf{r_2} + \mathbf{r_3} = 0$$
$$CE: \theta_{3b} = \theta_3 + \alpha_3$$

(solve by inspection using the given values above)

Step 2: List velocity unknowns:

$$\dot{r}_1, \dot{\theta}_3, \dot{\theta}_{3b}, \mathbf{V_p};\ \dot{r}_2, = input$$

five unknowns - need five equations.

Step 3: Identify and write equations:

$$\frac{d}{dt}(L1): -\dot{r}_1 e^{i\theta_1} + \dot{r}_2 e^{i\theta_2} + r_3\dot{\theta}_3 i e^{i\theta_3} = 0$$

2 equations from loop vector equation

$$\frac{d}{dt}(CE): \dot{\theta}_{3b} = \dot{\theta}_3$$

1 equation from constraint equation

$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}(\mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_{3b}) = r_2\dot{\theta}_2 i e^{i\theta_2} + r_3\dot{\theta}_3 i e^{i\theta_3} + r_{3b}\dot{\theta}_{3b} i e^{i\theta_{3b}}$$

2 equations of a vector chain from ground to point P

Total = 5 equations

Step 4: Expand equations into scalar form:

$$\frac{d}{dt}(L1): -\dot{r}_1 e^{i\theta_1} + \dot{r}_2 e^{i\theta_2} + r_3\dot{\theta}_3 i e^{i\theta_3} = 0$$

$$L1_x: -\dot{r}_1 c_1 + \dot{r}_2 c_2 - r_3\dot{\theta}_3 s_3 = 0$$

$$L1_y: -\dot{r}_1 s_2 + \dot{r}_2 s_2 + r_3\dot{\theta}_3 c_3 = 0$$

$$CE: \dot{\theta}_{3b} = \dot{\theta}_3$$

$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}(\mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_{3b}) = \dot{r}_2 e^{i\theta_2} + r_3\dot{\theta}_3 i e^{i\theta_3} + r_{3b}\dot{\theta}_{3b} i e^{i\theta_{3b}}$$

$$V_{px} = +\dot{r}_2 c_2 - r_3\dot{\theta}_3 s_3 - r_{3b}\dot{\theta}_{3b} s_{3b}$$

$$V_{py} = +\dot{r}_2 s_2 + r_3\dot{\theta}_3 s_3 + r_{3b}\dot{\theta}_{3b} c_{3b}$$

Step 5: Solve:

$$L1_x: -\dot{r}_1 * \cos(180) + 250 * \cos(-90) - 200 * \dot{\theta}_3 \sin(150) = 0$$

$$L1_y: : -\dot{r}_1 * \sin(180) + 250 * \sin(-90) + 200 * \dot{\theta}_3 * \cos(150) = 0$$

$L1_x: +\dot{r}_1 + 0 - 200 * \dot{\theta}_3 * .5 = 0$-→$\dot{r}_1 = -144.3$

$L1_y: : -\dot{r}_1 * 0 - 250 - 173.2 * \dot{\theta}_3 = 0$→ $\dot{\theta}_3 = -1.443$

$CE: \dot{\theta}_{3b} = \dot{\theta}_3$ → $\dot{\theta}_{3b} = -1.443$

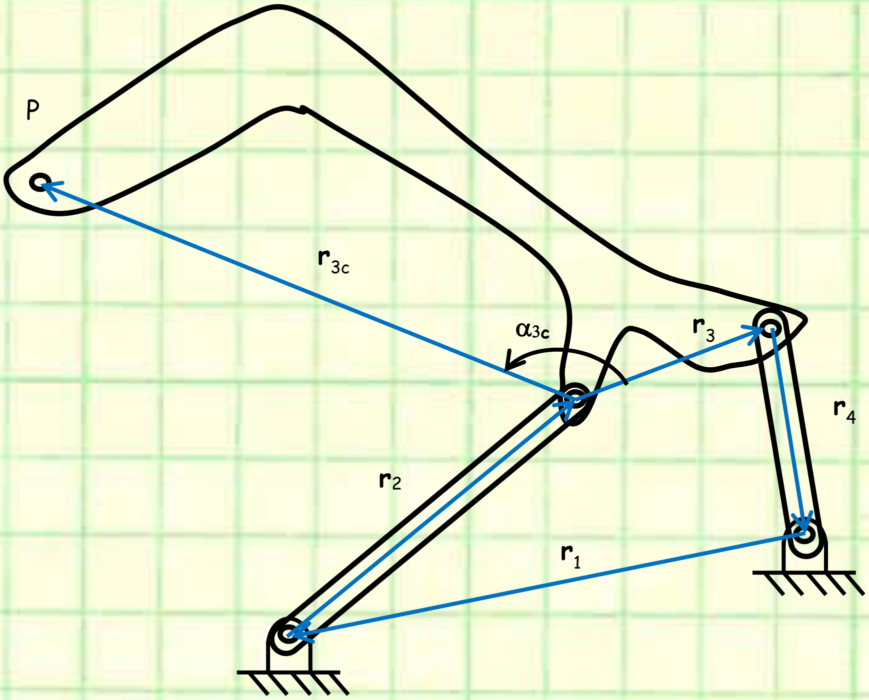
$$V_{px} = 250 * \cos(-90) - (200 + 300) * -1.443 * \sin(150) = 360.75$$

$$V_{py} = 250 * \sin(-90) + (200 + 300) * -1.443 * \cos(150) = 374.84$$

Discussion:

The results indicate that point P on the P is moving up and to the right. Two important notes:
1) The solution is linear in the velocities. This means that if the input velocity were doubled, the output velocity would double.
2) This solution is valid only at the position shown in the figure above. As the trebuchet moves, this solution is no longer valid, the angles need to be updated. This is demonstrated in example 3.

Example 3: Bobcat 650S loader: http://www.youtube.com/watch?v=aqK0qsXH3e4
Consider again the Bobcat 650 S loader with a simplifed version that ignores the bucket and input cyliner as shown in the figure below. This example will now consider an arbitary orientation for links 1-4. Given the fixed link lengths, input angle of link 2, input angular velocity of link 2, set up the equations to solve for the velocity unknowns in the model equations, and for the velocity of P. Describe a solution procedure for these equations.



Solution:

Step 1: get the position solution:

1. Schematic (see above)
2. Mobility: $M = 3(4-1) - 2(4) - 0 = 1$
3. Vector model (see above)
4. position unknowns $\theta_3, \theta_{3p}, \theta_4;\ \theta_2, = input$
5. Equations

$$L1: \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = 0$$
$$CE: \theta_{3p} = \theta_3 + \alpha_{3p}$$

Step 2: List velocity unknowns:

$$\dot{\theta}_3, \dot{\theta}_{3p}, \dot{\theta}_4, \mathbf{V}_{\boldsymbol{p}};\ \dot{\theta}_2 = input$$

five unknowns - need five equations.

Step 3: Identify and write equations:

$$\frac{d}{dt}(L1): r_2\dot{\theta}_2 ie^{i\theta_2} + r_3\dot{\theta}_3 ie^{i\theta_3} + r_4\dot{\theta}_4 ie^{i\theta_4} = 0$$

2 equations from loop vector equation

$$\frac{d}{dt}(CE): \dot{\theta}_{3p} = \dot{\theta}_3$$

1 equation from constraint equation

$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}\left(\mathbf{r}_2 + \mathbf{r}_{3p}\right) = r_2\dot{\theta}_2 ie^{i\theta_2} + r_{3p}\dot{\theta}_{3p} ie^{i\theta_{3p}}$$

2 equations of a vector chain from ground to point P

Total = 5 equations

Step 4: Expand equations into scalar form:

$$\frac{d}{dt}(L1): r_2\dot{\theta}_2 ie^{i\theta_2} + r_3\dot{\theta}_3 ie^{i\theta_3} + r_4\dot{\theta}_4 ie^{i\theta_4} = 0$$
$$L1_x: - r_2\dot{\theta}_2 s_2 - r_3\dot{\theta}_3 s_3 - r_4\dot{\theta}_4 s_4 = 0$$
$$L1_y: + r_2\dot{\theta}_2 c_2 + r_3\dot{\theta}_3 c_3 + r_4\dot{\theta}_4 c_4 = 0$$
$$CE: \dot{\theta}_{3p} = \dot{\theta}_3$$
$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}\left(\mathbf{r}_2 + \mathbf{r}_{3p}\right) = r_2\dot{\theta}_2 ie^{i\theta_2} + r_{3p}\dot{\theta}_{3p} ie^{i\theta_{3p}}$$
$$V_{px} = - r_2\dot{\theta}_2 s_2 - r_{3p}\dot{\theta}_{3p} s_{3p}$$
$$V_{py} = r_2\dot{\theta}_2 c_2 + r_{3p}\dot{\theta}_{3p} c_{3p}$$

Step 5: Write the equations in linear form:

This is personal preference, but I like to solve the model velocity unknowns in one matrix, and then follow up with equations to solve for V_p. That is what we will do here:

For the model, three equations (L1x, L1y, CE) and three unknowns ($\dot{\theta}_3, \dot{\theta}_{3p}, \dot{\theta}_4$), set up the equations in the matrix form, **Av** = **b**

Let the rows be the equations, columns be the unknowns:

$$\begin{bmatrix} -r_3 s_3 & -r_4 s_4 & 0 \\ +r_3 c_3 & -r_4 s_4 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_{3p} \end{Bmatrix} = \begin{Bmatrix} r_2 \dot{\theta}_2 s_2 \\ -r_2 \dot{\theta}_2 c_2 \\ 0 \end{Bmatrix}$$

And

$$V_{px} = -\, r_2 \dot{\theta}_2 s_2 - r_{3p} \dot{\theta}_{3p} s_{3p}$$
$$V_{py} = \ r_2 \dot{\theta}_2 c_2 + r_{3p} \dot{\theta}_{3p} c_{3p}$$

Step 6: Solution approach:
First, solve the matrix equation as: **v** = **A**$^{-1}$**b**

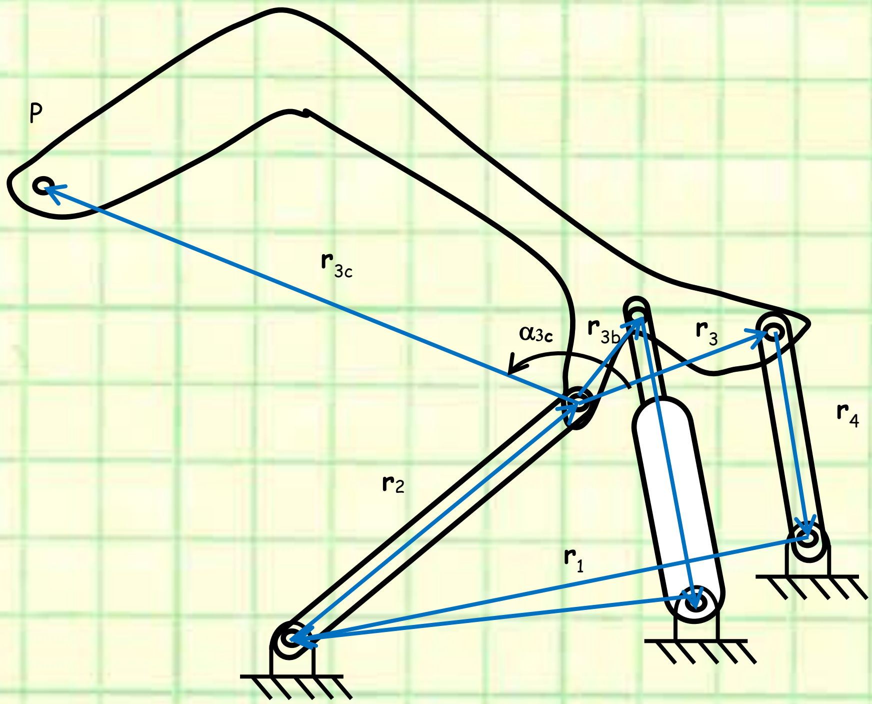
$$\dot{\theta}_3 = \mathbf{v}(1), \dot{\theta}_4 = \mathbf{v}(2), \dot{\theta}_{3p} = \mathbf{v}(3)$$

This gives all the values needed to solve for V_{px}, V_{py}.

Discussion:
The results require a matrix inverse to solve and therefore use of calculator, excel, Matlab or a C program.
1) The solution is linear in the velocities as shown by the linear equations. This means that if the input velocity were doubled, the output velocity would double.
2) These solution equations are valid at any position of the mechanism (except those when **A** is invertible).
3) pseudo code for a program to evaluate the velocity over range of motion on the skid steer might look like this:

- Start
- Clear workspace
- Define constant parameters (link lengths, etc)
- Define input velocity
- Loop through input displacement from min to max
    - For input displacement_i
    - Solve position
    - Define A, b matrix/vector
    - Solve velocity unknowns
    - Store results (Vpx, Vpy) in array
- End loop through displacement
- Plot results (plot(Vpx, Vpy)

Example 4: Bobcat 650S loader: http://www.youtube.com/watch?v=aqK0qsXH3e4
Consider now the Bobcat 650 S loader with a model that includes the input cylinder but ignores the bucket as shown in the figure below. This example will consider an arbitary orientation for links 1-4. Given the fixed link lengths, input angle of link 2, input angular velocity of link 2, set up the equations to solve for the velocity unknowns in the model equations, and for the velocity of P. Describe a solution procedure for these equations.



Solution:
Step 1: get the position solution:
1. Schematic (see above)
2. Mobility: $M = 3(4-1) - 2(4) - 0 = 1$
3. Vector model (see above)
4. position unknowns $\theta_3, \theta_{3p}, \theta_4$; $\theta_2, = input$
5. Equations

$$L1: \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = 0$$
$$L2: \mathbf{r}_{1b} + \mathbf{r}_2 + \mathbf{r}_{3b} + \mathbf{r}_5 = 0$$
$$CE: \theta_{3p} = \theta_3 + \alpha_{3p}, \theta_{3b} = \theta_3 + \alpha_{3b}$$

Step 2: List velocity unknowns:

$$\dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_{3p}, \dot{\theta}_{3b}, \dot{\theta}_4, \dot{\theta}_5, \mathbf{V}_{\boldsymbol{p}};\ \dot{r}_2 = input$$

eight unknowns - need eight equations.
Step 3: Identify and write equations:

$$\frac{d}{dt}(L1): r_2\dot{\theta}_2 ie^{i\theta_2} + r_3\dot{\theta}_3 ie^{i\theta_3} + r_4\dot{\theta}_4 ie^{i\theta_4} = 0$$
$$\frac{d}{dt}(L2): r_2\dot{\theta}_2 ie^{i\theta_2} + r_{3b}\dot{\theta}_{3b} ie^{i\theta_{3b}} + \dot{r}_5 e^{i\dot{\theta}_5} + r_5\dot{\theta}_5 ie^{i\theta_5} = 0$$

4 equations from loop vector equation

$$\frac{d}{dt}(CE): \dot{\theta}_{3p} = \dot{\theta}_3, \dot{\theta}_{3b} = \dot{\theta}_3$$

2 equations from constraint equation

$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}\left(\mathbf{r}_2 + \mathbf{r}_{3p}\right) = r_2\dot{\theta}_2 ie^{i\theta_2} + r_{3p}\dot{\theta}_{3p} ie^{i\theta_{3p}}$$

2 equations of a vector chain from ground to point P

Total = 8 equations

Step 4: Expand equations into scalar form:

$$\frac{d}{dt}(L1): r_2\dot{\theta}_2 ie^{i\theta_2} + r_3\dot{\theta}_3 ie^{i\theta_3} + r_4\dot{\theta}_4 ie^{i\theta_4} = 0$$

$$L1_x: -r_2\dot{\theta}_2 s_2 - r_3\dot{\theta}_3 s_3 - r_4\dot{\theta}_4 s_4 = 0$$

$$L1_y: +r_2\dot{\theta}_2 c_2 + r_3\dot{\theta}_3 c_3 + r_4\dot{\theta}_4 c_4 = 0$$

$$\frac{d}{dt}(L2): r_2\dot{\theta}_2 ie^{i\theta_2} + r_{3b}\dot{\theta}_{3b} ie^{i\theta_{3b}} + \dot{r}_5 e^{i\dot{\theta}_5} + r_5\dot{\theta}_5 ie^{i\theta_5} = 0$$

$$L2_x: -r_2\dot{\theta}_2 s_2 - r_{3b}\dot{\theta}_{3b} s_{3b} + \dot{r}_5 c_5 - r_5\dot{\theta}_5 s_5 = 0$$

$$L2_y: r_2\dot{\theta}_2 c_2 + r_{3b}\dot{\theta}_{3b} c_{3b} + \dot{r}_5 s_5 + r_5\dot{\theta}_5 c_5 = 0$$

$$CE: \dot{\theta}_{3p} = \dot{\theta}_3, \dot{\theta}_{3b} = \dot{\theta}_3$$

$$\mathbf{V}_{\boldsymbol{p}} = \frac{d}{dt}\left(\overrightarrow{OP}\right) = \frac{d}{dt}\left(\mathbf{r}_2 + \mathbf{r}_{3p}\right) = r_2\dot{\theta}_2 ie^{i\theta_2} + r_{3p}\dot{\theta}_{3p} ie^{i\theta_{3p}}$$

$$V_{px} = -r_2\dot{\theta}_2 s_2 - r_{3p}\dot{\theta}_{3p} s_{3p}$$

$$V_{py} = r_2\dot{\theta}_2 c_2 + r_{3p}\dot{\theta}_{3p} c_{3p}$$

Step 5: Write the equations in linear form:
This is personal preference, but I like to solve the model velocity unknowns in one matrix, and then follow up with equations to solve for V_p. That is what we will do here:

For the model, three equations (L1x, L1y, CE) and three unknowns ($\dot{\theta}_3, \dot{\theta}_{3p}, \dot{\theta}_4$), set up the equations in the matrix form, **Av** = **b**
Let the rows be the equations, columns be the unknowns:

$$\begin{bmatrix} -r_2 s_2 & -r_3 s_3 & 0 & 0 & -r_4 s_4 & 0 \\ r_2 c_2 & r_3 c_3 & 0 & 0 & r_4 c_4 & 0 \\ -r_2 s_2 & 0 & -r_{3b} s_{3b} & 0 & 0 & -r_5 s_5 \\ r_2 c_2 & 0 & r_{3b} c_{3b} & 0 & 0 & r_5 c_5 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_{3b} \\ \dot{\theta}_{3p} \\ \dot{\theta}_4 \\ \dot{\theta}_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -\dot{r}_5 c_5 \\ -\dot{r}_5 s_5 \\ 0 \\ 0 \end{Bmatrix}$$

And

$$V_{px} = -r_2\dot{\theta}_2 s_2 - r_{3p}\dot{\theta}_{3p} s_{3p}$$

$$V_{py} = r_2\dot{\theta}_2 c_2 + r_{3p}\dot{\theta}_{3p} c_{3p}$$

Step 6: Solution approach:
First, solve the matrix equation as: **v** = $\mathbf{A}^{-1}\mathbf{b}$

$$\dot{\theta}_2 = \mathbf{v}(1), \dot{\theta}_3 = \mathbf{v}(2), \dot{\theta}_{3b} = \mathbf{v}(3), etc$$

This gives all the values needed to solve for V_{px}, V_{py}.

<u>Discussion:</u>

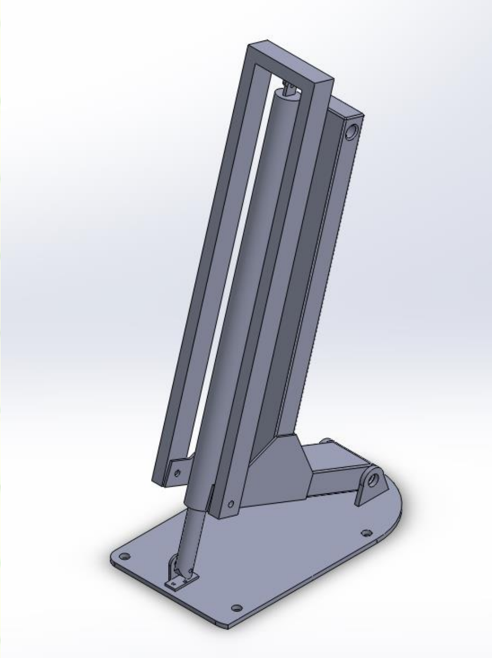
The results require a matrix inverse to solve and therefore use of calculator, excel, Matlab or a C program.

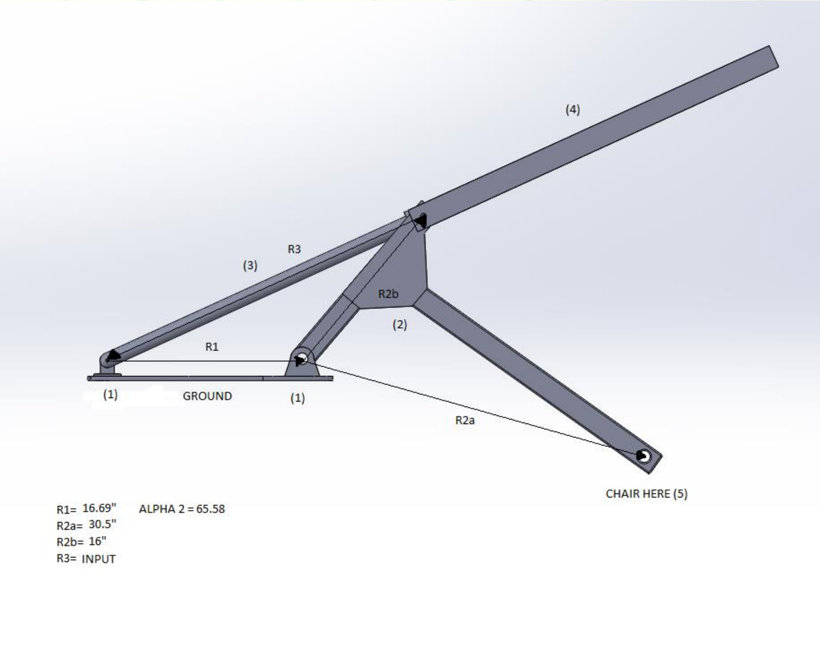
1) The solution is linear in the velocities as shown by the linear equations. This means that if the input velocity were doubled, the output velocity would double.

2) These solution equations are valid at any position of the mechanism (except those when **A** is invertible).

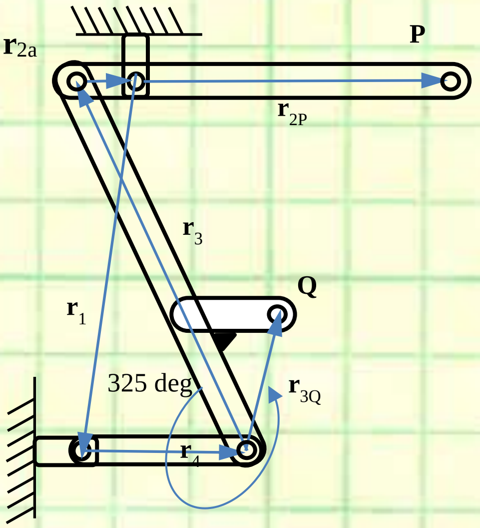
3) the position problem is a little harder. As a recommendation, solve assuming that theta2 is the input displacement, for a full range of motion.

Example 5:
(Taken from the Pool-lift team, F12)





Velocity Analysis: Given the exercise mechanism shown in the picture below with a simplified schematic, vector and position model. The rotation rate of the handle, link 2 (theta2_dot = 1 rad/s) is given along with other link lengths and orientations. Solve for the velocity of point Q attached to the seat as shown, as well as the other unknown velocities in the model. Use the method covered in class and show all your work.



$$\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} = 0$$
$$\theta_{3Q} = \theta_3 + 325$$

r_1 = 510 cm; r_2 = 120 cm; r_3 =538 cm; r_4 = 300 cm, r_{2P} = 500 cm; r_{3Q} = 150 cm;
θ_1 = 250 deg; θ_2 = 0 deg; θ_3 = 117 deg; θ_4 = 0 deg; branch = -1
θ_{2_dot} = 1 rad/s

3) Velocity analysis: Classical techniques (instant centers, centrodes, etc.)

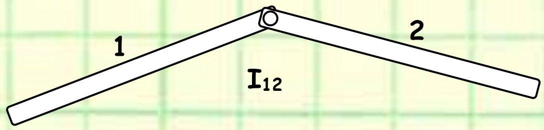
Classical techniques for velocity analysis consists predominantly of graphical techniques and determination of instant centers. The graphical techniques involved drawing velocity polygons that form geometric equivalents of our derivative loop closure equations. Due to the fact that analytical techniques can be easily programmed and formalized, graphical techniques have been largely outdated. However, some of these techniques provide a significant amount of insight into the problem and will be reviewed briefly here. The techniques reviewed are:

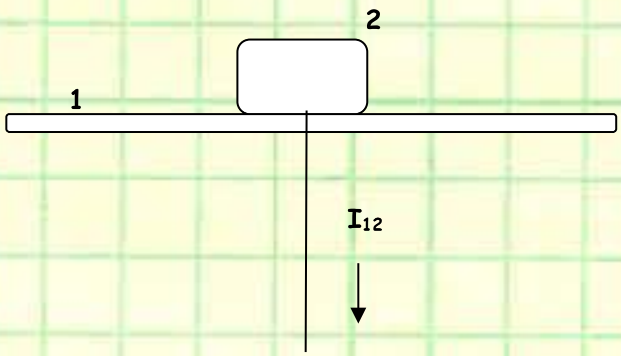
- Instant Centers
- Centrodes

<u>Instant Centers of Velocity</u> or instant centers are a point common to two bodies which has the same velocity for both bodies (at that instant in time). The number of instant centers for an *n*-body linkage is:

$$c=n*(n-1)/2.$$

The instant center of two links connected by a revolute is trivial (it is that revolute). The instant center for two links connected by a slider is also simple (The center of curvature of the slider axis).





For bodies not connected immediately by a joint, the technique relies on **Kennedy's Theorem:**

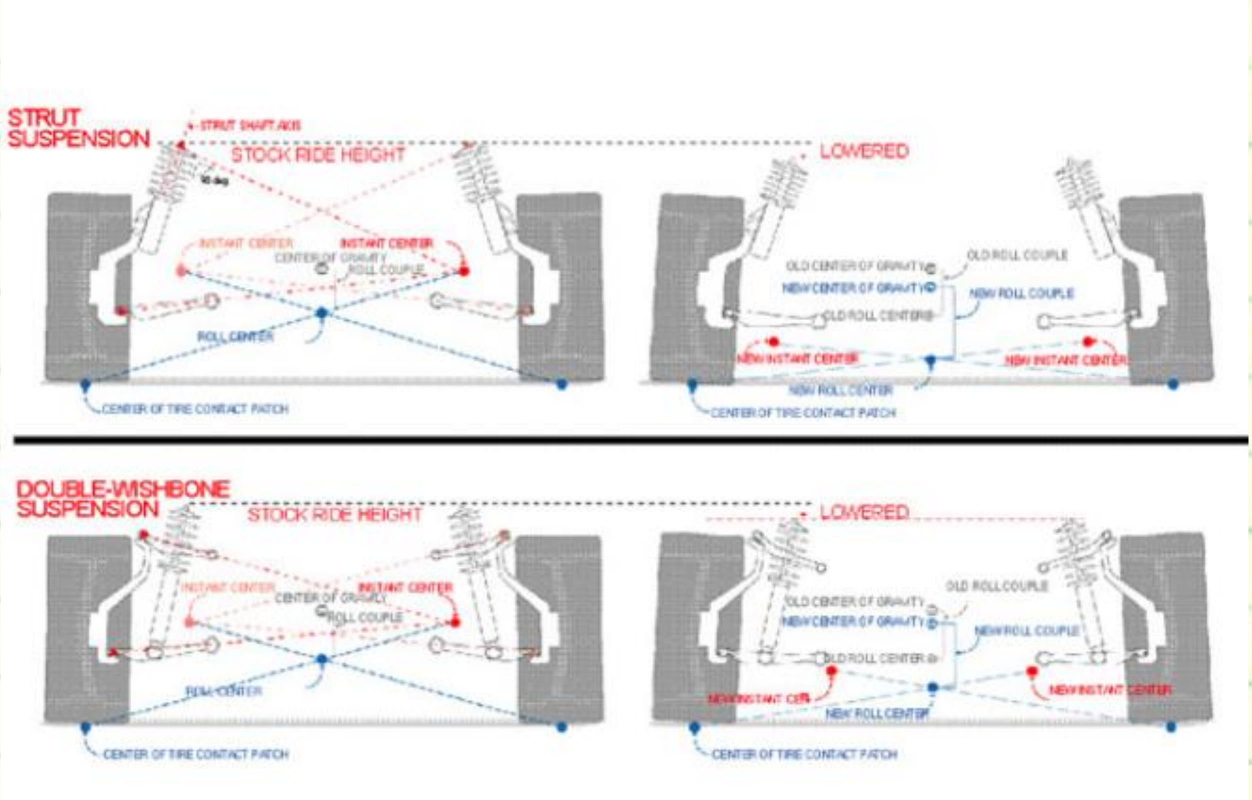
Kennedy's Theorem: *Any three bodies in plane motion will have three instant centers and they will lie on the same straight line.*

Applying Kennedy's theorem to a four bar:

Applying Kennedy's theorem to a Slider-crank:

A few examples of the use of instant centers:

Vehicle Suspension Design:



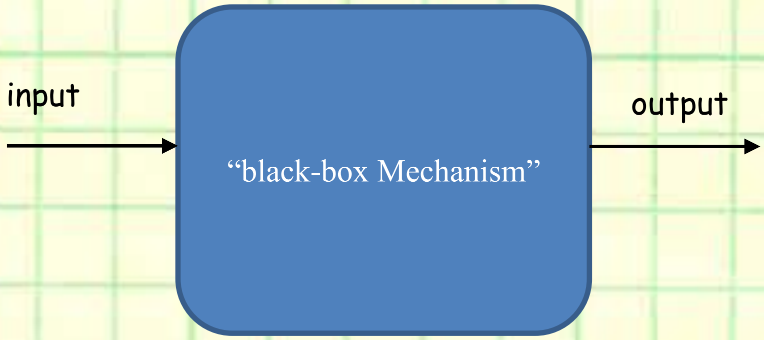
Centrodes:

A Centrode is the curve defined by the locations of the instant center over the range of motion of a mechanism. Each possible instant center can create two centrodes, found by considering the motion relative to each of the two links defining the instant center. One centrode will be called the fixed centrode and one a moving centrode. The centrodes can then recreate the motion of the fourbar by rolling (without slip) in contact with each other. As an example, fourbars can be used to define the profile for non-circular gears (for example elliptical gears).

Simple KinetoStatic force analysis assuming low dynamic effects:

Kinetostatic force analysis provides a direct method to solve for forces in machines. In this method, we will assume dynamic effects in the mechanism are minimal. Thus, we can solve for the forces in a mechanism based the velocity analysis. This process is based on the principles of conservation of energy (or power here since we assume the constraints are not time-dependent) and superposition.

Consider the mechanism as a black box as shown in the figure below. Force and motions are applied at the input and force and motion occurs at the output



The following assumptions apply to this system:
1) Problem is treated in an instantaneous sense
2) Ignore dynamic effects in the mechanism
3) System is conservative - Energy is not stored or created in the mechanism
4) Mechanism efficiency is given as η

Thus, the input power equals the output power:

$$P_{in} = P_{out}$$

With the power instantaneously defined as,

$$P = \mathbf{F} \cdot \mathbf{v}$$

For linear forces/velocity

$$P = \mathbf{T} \cdot \boldsymbol{\omega}$$

For rotational forces/velocity

Further, given

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$
$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

And, from triple scalar product:
a.bxc = b.cxa = c.axb

It can be shown these are equal statements.

A relationship between input and output motion and force is then given as:

If we assume that the $\mathbf{v}_{in}$ is found in the direction of the input force and similarly, $\mathbf{v}_{o}$ is found in the direction of the output force, the equation can be rewritten as:

(with * reminding that a particular velocity component is used)

This is sometimes expressed as the Mechanical Advantage, defined as:

For linear systems or

For rotational systems

Application: Method of Virtual work
More generally, we will call this the method of virtual work, (in virtual work, we ignore time-varying constraints) in which all the energies are summed on one side of the equation and are set equal to zero (no energy is created or stored):

$$\mathbf{F}_{\boldsymbol{i}} \cdot \mathbf{v}_{\boldsymbol{i}} + \mathbf{T}_{\boldsymbol{j}} \cdot \boldsymbol{\omega}_{\boldsymbol{j}} = 0$$

(Dynamic forces can be inserted into the virtual work equation).

A few notes:
1) These are EXTERNAL forces and torques (internal forces / torques do no work)
2) I prefer to use the virtual work equation:
3) There is a slight difference in Virtual work and Mechanical Advantage: In Virtual work, the external forces and torques are acting on the mechanism. In Mechanical advantage, the input is acting on the mechanism, the output is the force of the mechanism on the output device.

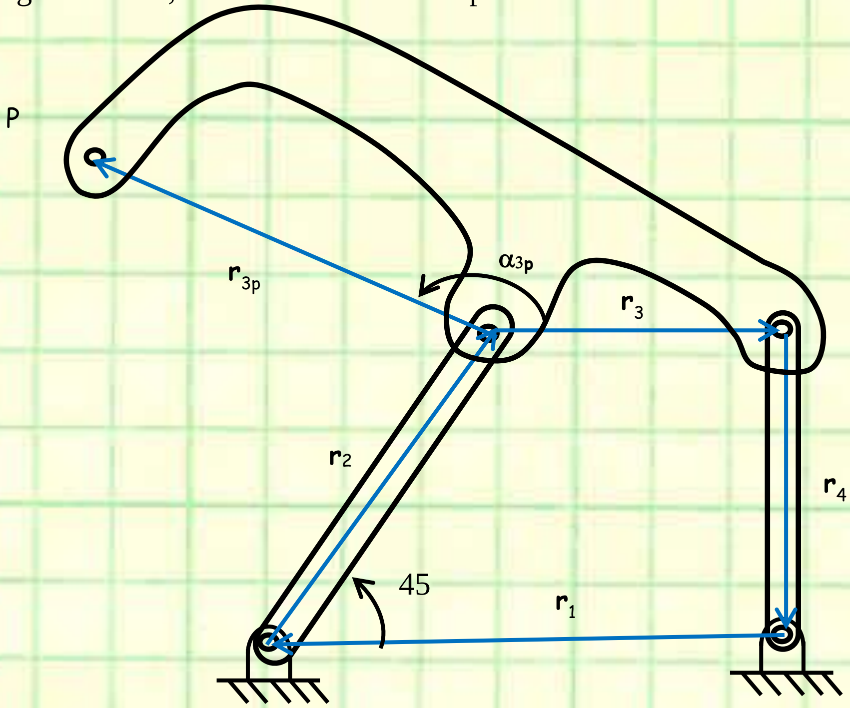
Extra notes Kinetostatic Force (w/o dynamics) Analysis:
To complete the kinetostatic force analysis, first apply the idea of mechanical advantage to evaluate the input force required for each applied load. If loads are given on multiple links, then evaluate the mechanical advantage for these multiple links. Second, apply the principle of

superposition (these problems are linear in force). Thus, the total input force is given as the superposition of the input forces required for each of the applied loads.

The last possible step to consider here is constraint forces (forces in the bearings). In line with the concept of static force analysis based on conservation of energy, one approach would be to repeat the process above for every bearing, but instantaneously eliminate the motion constraint from each bearing, and solve for the force required to enforce this constraint. For example, to find the x-component reaction of a bearing, allow that bearing to move (assign it unit velocity) instantaneously (i.e., bearing does not change position). Solve for the mechanical advantage relating the x-force at that bearing to all applied loads, and sum to get the total x-directed reaction via superposition. This method is known as the method of Lagrange Multipliers. If only one or two reactions are desired, it is relatively easy to apply. If all reactions are desired, it makes more sense to apply the techniques of kinetostatic analysis (to be covered in upcoming topics).

Example 1: Focres in Bobcat 650S loader: http://www.youtube.com/watch?v=aqK0qsXH3e4

Recondsider the simplied model from the velocity example that ignores the bucket, ignores the input cylinder, and assumes that links 1 and 3 are horizontal, link 4 is vertical and link 2 is at 45 degrees. Also, assume link 2 is the input.



Given r_1 = 161 cm; r_2 = 141 cm, r_3 = 120 cm, r_4 = 100 cm, θ_2 = 45deg, $\dot{\theta}_2$ = .5 rad/s, r_{3p} = 200, α_{3p} = 135deg, find the input torque required at link 2 to lift a 1000 kg. load placed at point P.

Solution:

Step 1: get the velocity solution:

$L1_x$: $-49.85 - 0 + 100 * \dot{\theta}_4 = 0 \rightarrow \dot{\theta}_4 = .4985$

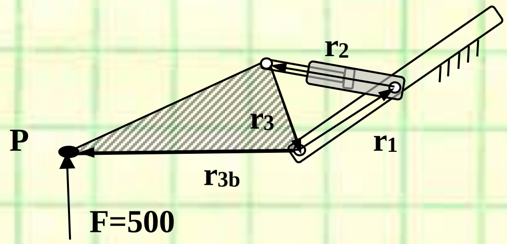
$L1_y$: $+49.85 + 120 * \dot{\theta}_3 + 0 = 0 \rightarrow \dot{\theta}_3 = -.4154$

CE: $\dot{\theta}_{3p} = \dot{\theta}_3 \rightarrow \dot{\theta}_{3p} = -.4154$

$$V_{px} = -141 * 0.5 * \sin(45) - 200 * -.4154 * \sin(135) = 8.896$$
$$V_{py} = +141 * 0.5 * \cos(45) + 200 * -.4154 * \cos(135) = 108.6$$

Step 5: Setup the conservation of power equations:

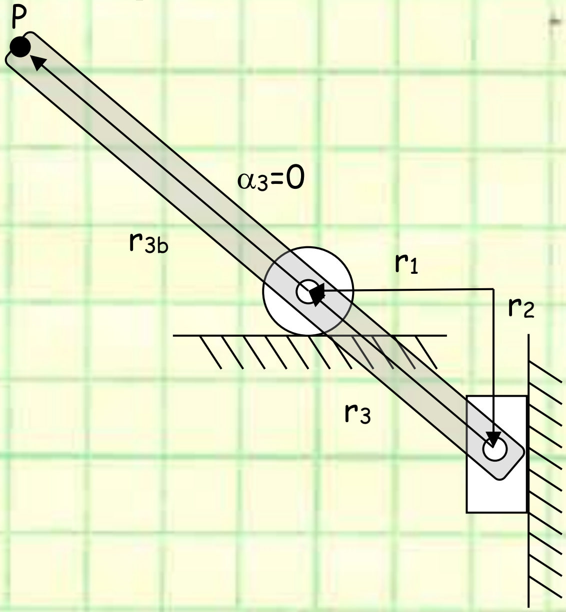
Example: Kinetic force analysis Consider the rear suspension mechanism shown on the bike below with schematic and vector model as defined. The velocity problem has also been solved with results given below. If a 500 N load is applied at the wheel axle (point P) in the vertical up direction, what is the force required in the spring-over-shock member (r_2) for equilibrium (force directed along the axis of the r_2).



r_1 = 20cm, r_2 =25 cm, r_3 = 35 cm,

θ_1=50 deg, θ_2=128.5 deg, θ_3=-85.6 deg, θ_{3b}=175 deg,

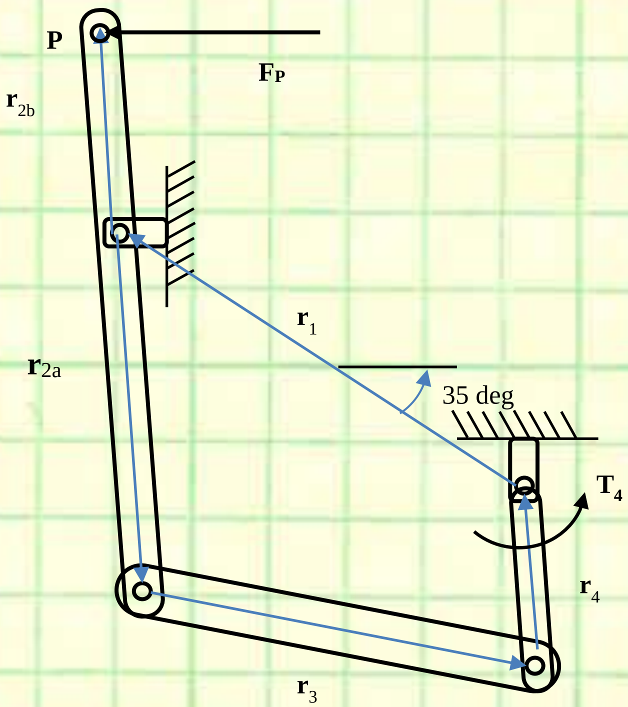
r_2_dot = 75cm/s, V_{px}=5 cm/s, V_{py}=125 cm/s, Given the floating arm trebuchet shown as a schematic below. Assume r_2, r_2_dot are the inputs, r_1 = 173cm, r_2 = 100cm, theta3 = 150 deg., r_3 = 200cm, r_{3b} 300 cm, and r_2_dot = 250 cm/s, assume a 100kg weight on the vertical slider, find the force at point P vertical to the bar.



From Velocity:

$$\dot{\theta}_{3b} = 1.443$$
$$V_{px} = 250 * \cos(-90) - (200 + 300) * -1.443 * \sin(150) = 360.75$$
$$V_{py} = 250 * \sin(-90) + (200 + 300) * -1.443 * \cos(150) = 374.84$$

Given the exercise mechanism shown in the figure below with a resistance torque on link 4, solve for the necessary input force applied at P horizontal and to the left to keep the machine operating at a rotational speed of θ_4_dot = +20rpm. Use the method covered in class and show all work.



T_4 = -100Ncm resistance torque, θ_4_dot = +20rpm, θ_{2b}_dot = +10rpm,

r_1 = 1150, r_{2a} =850, r_{2b} =450, θ_{2b} = 100 deg, r_3 = 950, r_4 = 400