Part IX: Force Analysis

This set of notes will discuss force analysis of mechanisms. The areas covered in this section are:

1) Introduction to Force Analysis
2) Review of Dynamics
3) Force Analysis methods for kinetostatics
4) Matrix Method
5) Method of Virtual Work

Introduction to force analysis
In dynamic force analysis, the approach is to create equations that relate forces and motion of the bodies in the mechanism. These are called equations of motion and are written as:

$$\mathbf{F} = m\mathbf{a}$$

There are 2 directions to these problems: Forward Dynamics, and Inverse Dynamics.
Forward Dynamics:

Inverse Dynamics;

Motivational points:

1. Performing force analysis of a mechanism draws on all your modeling skills

2. Analysis that leads to deflection / stress determinations critical for design

3. This basic skill is needed to pass this class.

Review of Dynamics:

1: Newtonian Mechanics: via 3 laws

1. Conservation of momentum:

2. Force = rate of change of momentum

3. Action/reaction

Key points:

2: Corollary: Euler's Equation

1. Torque = rate of change of angular momentum:

2. For 2 D bodies:

Summing moments about D: (I about c.g.)

Summing moments about c.g.:

Other points to review:

1. 2 force member

Kinetostatic Analysis Techniques:
1. Superposition

2. Matrix Method

3. Method of Virtual Work

Force Analysis Using the Matrix Method:
In the matrix method, equations of dynamic equilibrium are written for FBD's of all the links in the mechanism with all forces included (this includes forces at the kinematic pairs). This results in a coupling of unknown forces
Equations are linear in the forces.

For example:

$$[\mathbf{C}]\begin{Bmatrix}\mathbf{F}\\ \mathbf{T}\end{Bmatrix} = \begin{Bmatrix}m\mathbf{a}\\ \mathbf{I}\boldsymbol{\alpha}\end{Bmatrix}$$

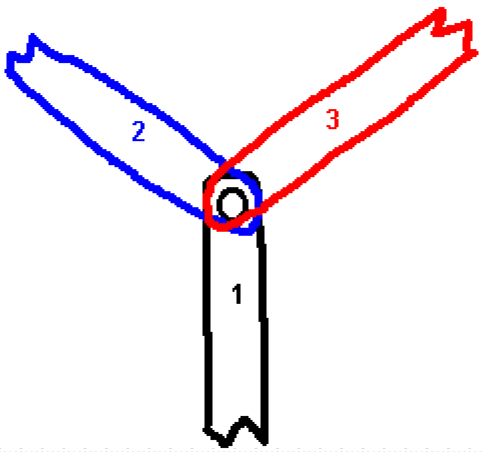
In general, there will be many unknowns → solve in computer program.

Steps:

1) Add vector labels on each body, vector from each joint to each body c.m, also from each point of force application to each c.m.
2) Construct FBD's of every body (except ground)
3) Write equations of equilibrium for each body – express as scalars
4) Write equations in matrix form
5) Solve for constraint forces, input forces.

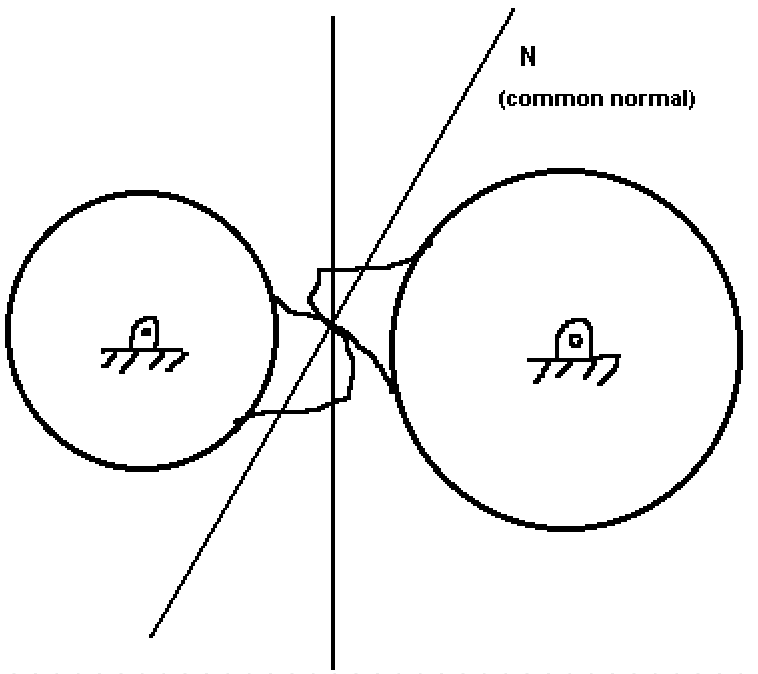
Some special cases you might see:

1. Multiple links at 1 joint:



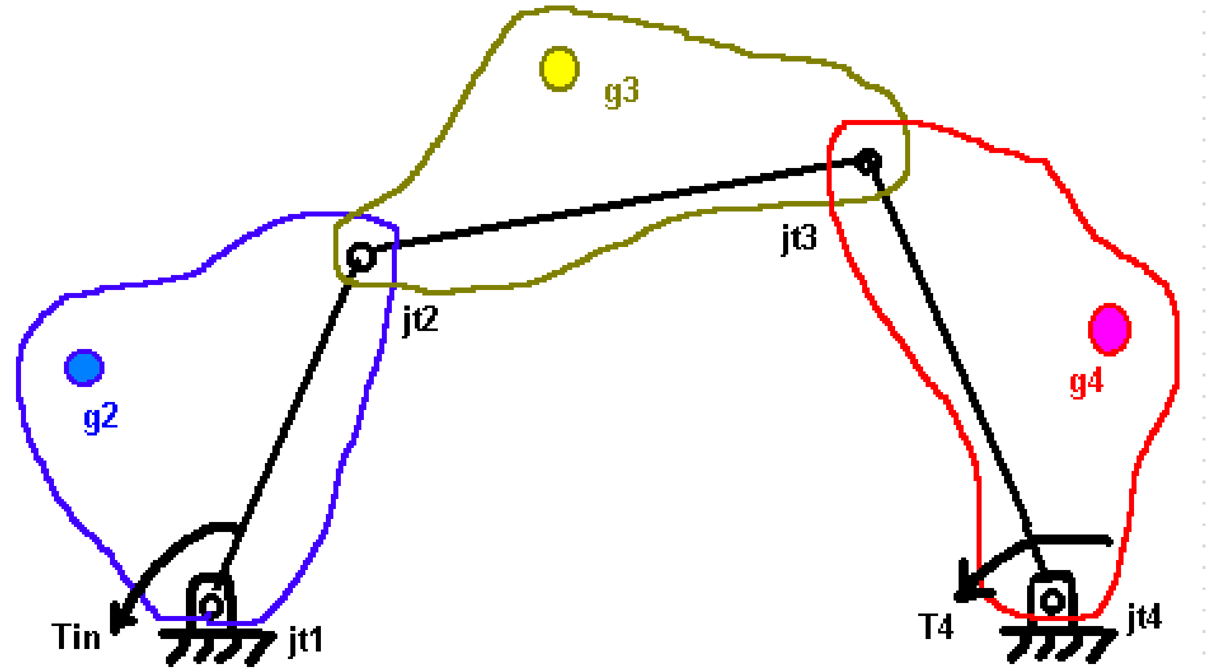
Treat as n-1 constraint forces (as in mobility)

2. Gears, cams:

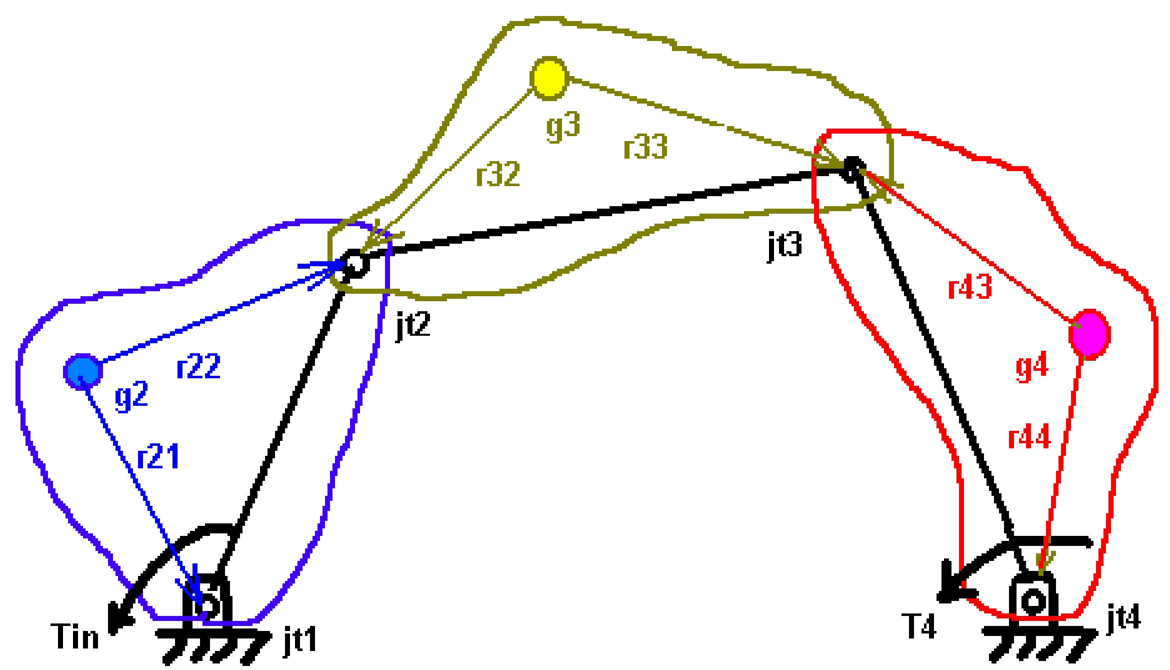


The force acts along the common normal (line of action) and is known.

Matrix Method: Example 1:
Consider this generic fourbar:



Step 1: Add labeling with following notation



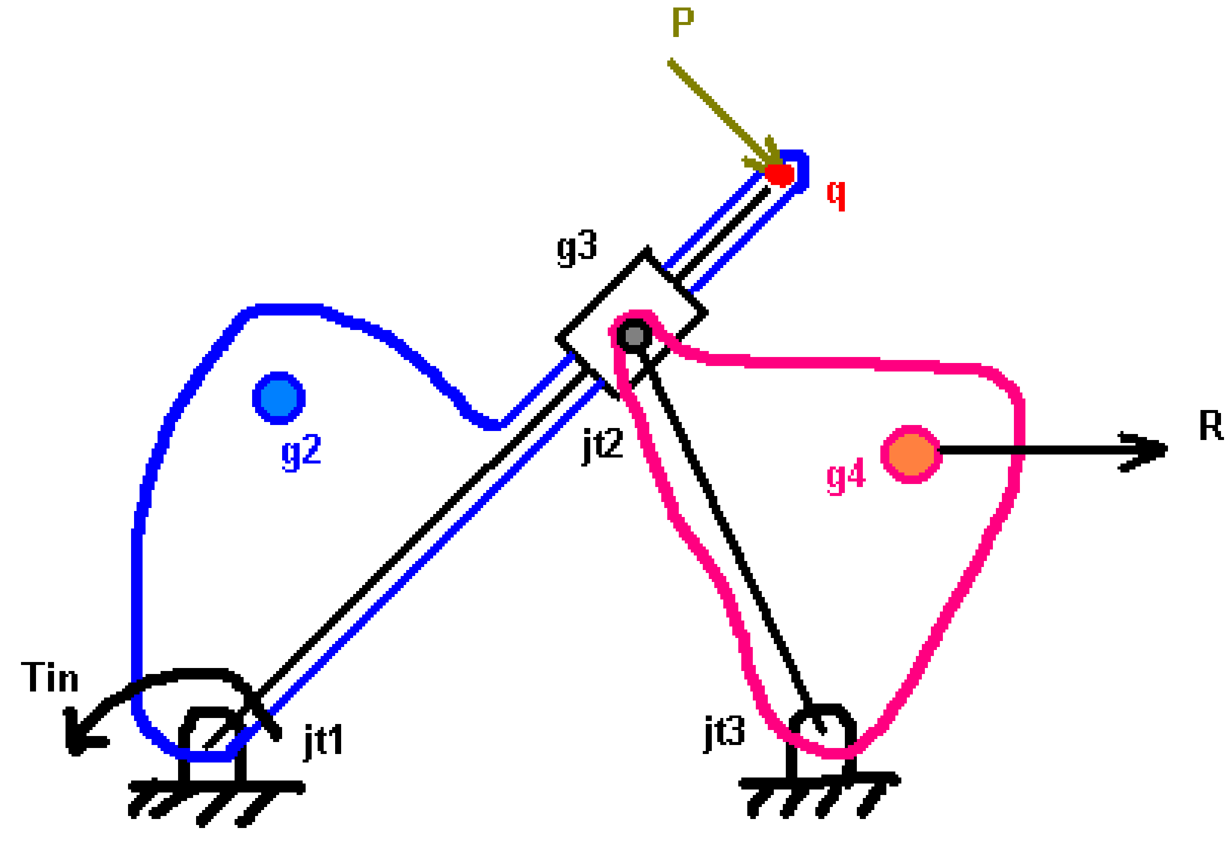
g_i = center of mass of link i
jt_i = joint i
r_{ij} = vector from cm of *I* to joint *j*
$\mathbf{F_{ij}}$ = force of body *i* on body *j*

Step 2: Construct FBD's of all bodies (except for ground)

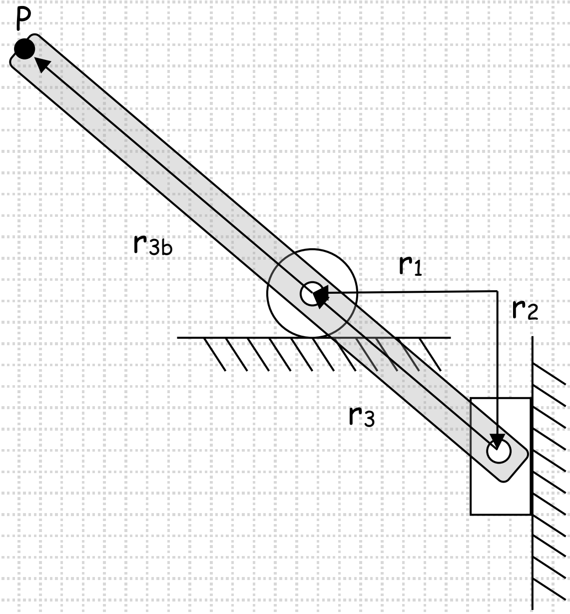
Step 3: Construct equations of dynamic equilibrium for all bodies (write in scalar form)

Step 4: rewrite equations in matrix form

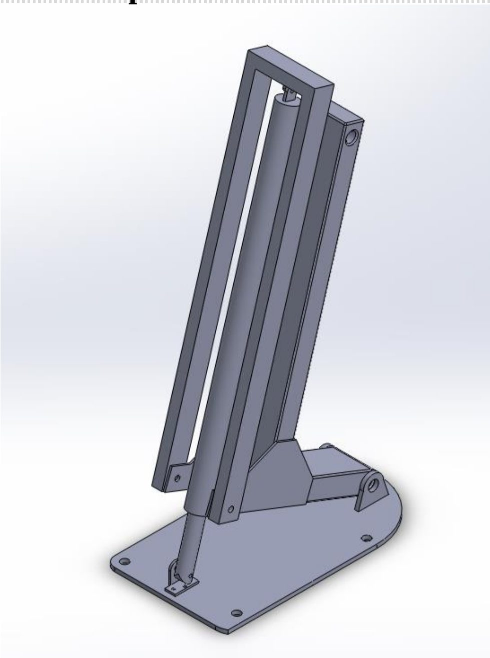
Example 2:

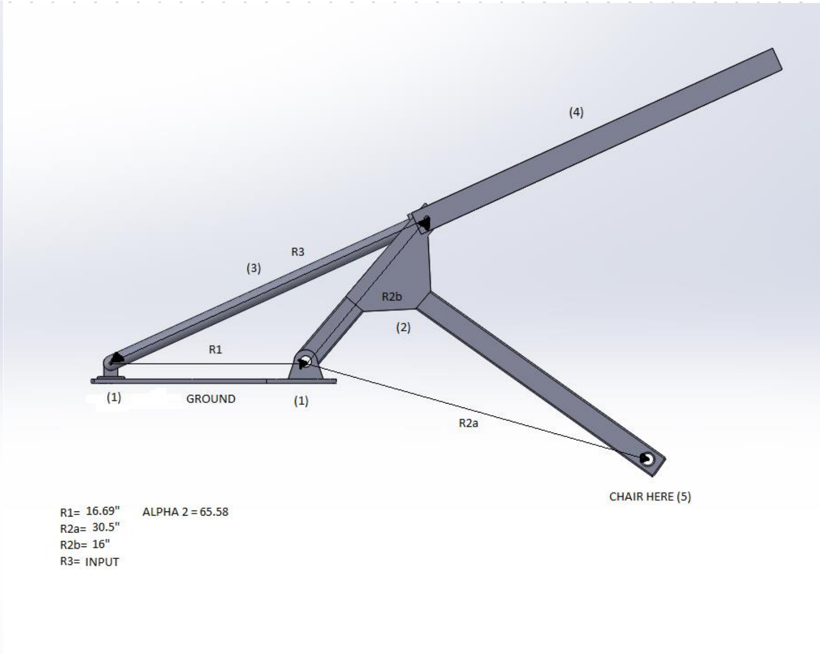


Example 3:



Example 4:





(4)
R3
(3)
R2b
(2)
R1
(1)
GROUND
(1)
R2a
CHAIR HERE (5)
R1= 16.69"
ALPHA 2 = 65.58
R2a= 30.5"
R2b= 16"
R3= INPUT

Force Analysis via the Method of Virtual Work:

1. "if a rigid body is in equilibrium under the action of external forces, the total work done by these forces is zero for a small displacement of the body"

$$\delta W = 0$$

2. Work:

$$W = \int \mathbf{F} \cdot d\mathbf{x} \text{ or } W = \int \mathbf{T} \cdot d\boldsymbol{\theta}$$

with **F, x, T, q,** vectors and *W* a scalar.

3. To indicate we are dealing with displacements that occur over an very small period of time (called virtual displacements), use the notation:

$$\delta W = \delta\mathbf{F} \cdot \delta\mathbf{x}, \; \delta W = \delta\mathbf{T} \cdot \delta\boldsymbol{\theta}$$

4. Now apply the virtual work definition:

$$\delta W = \sum_i \delta\mathbf{F}_{\boldsymbol{i}} \cdot \delta\mathbf{x}_{\boldsymbol{i}} + \sum_j \delta\mathbf{T}_{\boldsymbol{j}} \cdot \delta\boldsymbol{\theta}_{\boldsymbol{j}} = 0$$

5. Divide this by a small time step and get,

$$\sum_i \mathbf{F}_{\boldsymbol{i}} \cdot \mathbf{v}_{\boldsymbol{i}} + \sum_j \mathbf{T}_{\boldsymbol{j}} \cdot \boldsymbol{\omega}_{\boldsymbol{j}} = 0$$

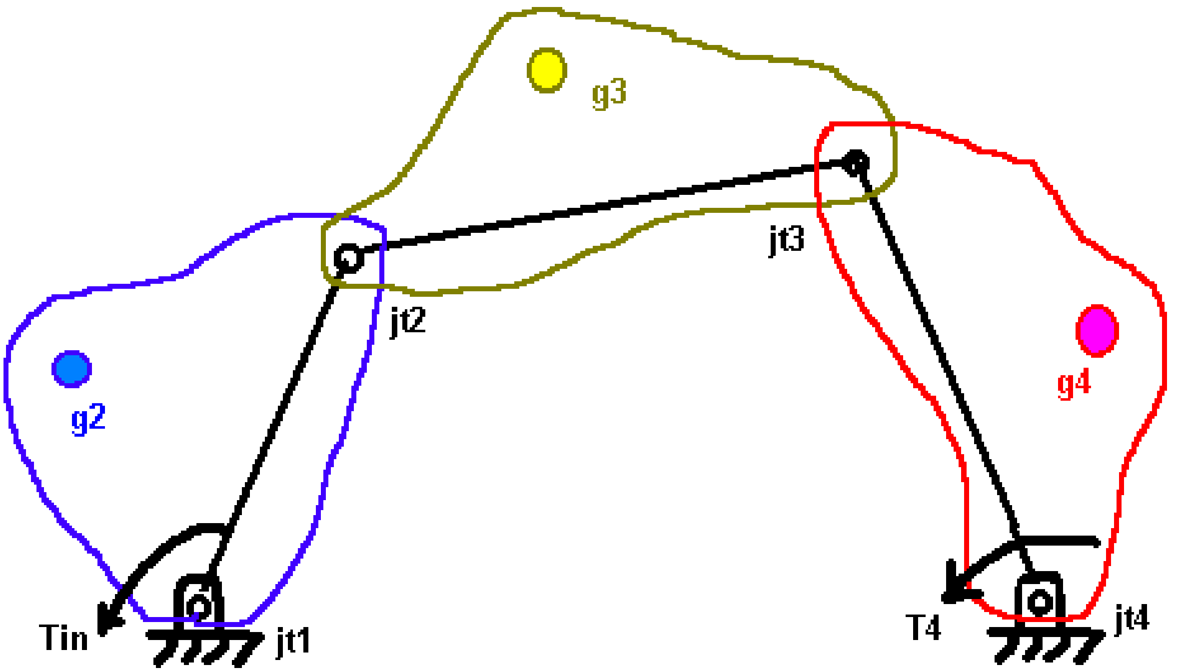
6. These are all the external torques and forces on the body including gravity. Use D'almebert's method to include inertial forces as,

$$\sum_i \mathbf{F}_{\boldsymbol{i}} \cdot \mathbf{v}_{\boldsymbol{i}} + \sum_j \mathbf{T}_{\boldsymbol{j}} \cdot \boldsymbol{\omega}_{\boldsymbol{j}} + \sum_k \mathbf{F}_{\boldsymbol{Ok}} \cdot \mathbf{v}_{\boldsymbol{k}} + \sum_l \mathbf{T}_{\boldsymbol{Ol}} \cdot \boldsymbol{\omega}_{\boldsymbol{l}} = 0$$

With

$$\mathbf{F}_{\boldsymbol{Ok}} = -m\boldsymbol{a}_{\boldsymbol{gk}}, \; \mathbf{T}_{\boldsymbol{Ok}} = -\mathbf{I}\boldsymbol{\alpha}_{\boldsymbol{gk}},$$

Example 1:



Body i	m_{gi}	v_{gix}	v_{giy}	a_{gix}	a_{giy}	ω_{gix}	α_{gix}	$I_{gi,z}$
Units	kg	m/s	m/s	m/s^2	m/s^2	rad/s	rad/s^2	kgm^2
2	10	3	4	-1	2	2	.1	1
3	5	6	-2	1	-1	1	.2	.5
4	12	3	1	-1	2	-.1	-.1	2

***** all numbers fictitious

Example 2: