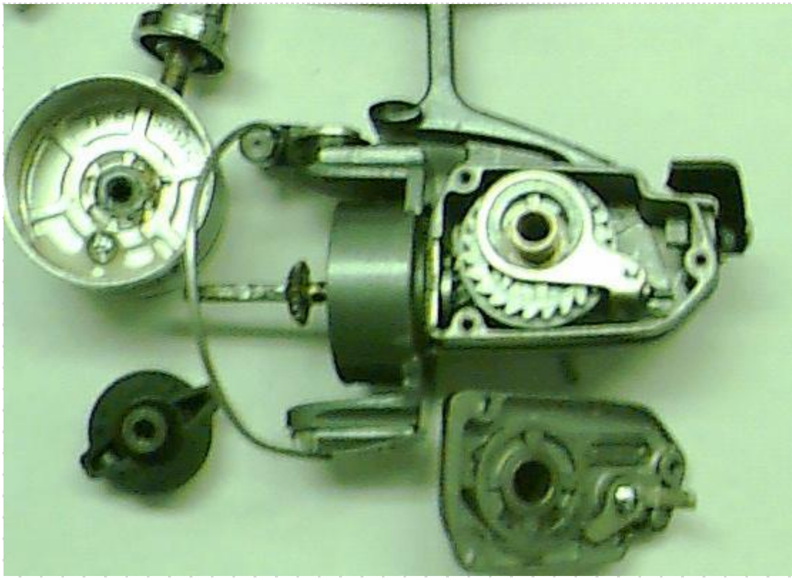
Part IV: Mechanism Analysis: modeling and position analysis

Section Topics:

1. Mechanism Analysis; purpose
2. Mechanism Analysis, procedure
3. Worked Example
4. Sample Mechanism examples



1. Mechanism Analysis: Given a mechanism, completely describe its motion.

Mechanism analysis is performed by first constructing a representative model of the mechanism that permits a ready mathematical description. The general process is outlined as follows:

1) Replace the physical elements (primarily bodies) of the mechanism with representative vector elements
2) Add appropriate constraints on these vectors as defined by the joints of the mechanism
3) Now that the vectors are coupled to each other via constraints, and since vectors are mathematical descriptors of physical objects, they can be combined using rules of vector algebra (for position analysis) and vector calculus (for velocity and acceleration analysis) to result in equations to be solved.
4) Thus, the model should imitate the behavior of the mechanism during analysis.

Note: Advanced models may include any variety of conditions such as friction, joint limits, interference, etc.

Purpose and Benefits of mechanism modeling

- Provides insight and intuition about the device
- Low cost of a model compared to actual device
- Flexibility
- Serves as a design tool
- Safer, cheaper, faster
- Better Final Results

2. Procedure

Step 1: Draw a schematic of the mechanism.

- This will clear identify the links and joints.

Step 2: Perform Mobility analysis.

- Determine the number of dof and input actuators.

Step 3: Create a vector model of the mechanism.

- By replacing all rigid bodies in the mechanism with appropriate, representative vectors.
- Note; kinematically, a link is defined as a rigid, straight-line connection between two joints.

Step 4: List the unknowns in the model

- The unknowns in the analysis model consist of all the variable parameters within the model, minus the number of inputs (dof)

Step 5: Write the position equations needed to solve.

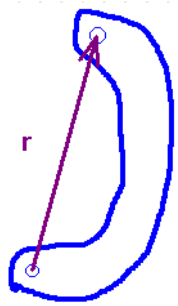
- There are 2 types of equations: loop closure equations (usually vector), and constraint equations (usually scalar).
- Be careful that loop equations written are unique.

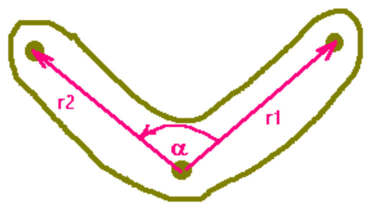
Step 6: Solve the equations

Schematic → Vector model: Typical cases you may in the process of creating the vector model:

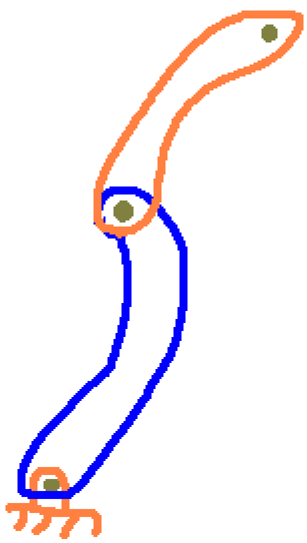
1) Binary links:
Replace with a single vector from one revolute to the next. Length is assumed known (since it is constant) and angle is unknown if it is not an input.

2) Ternary links:
Two vectors (of constant and assumed known) length will represent this link. The vectors are separated by a constant angle (since it is fixed, it is assumed to be known). Thus, there is only one unknown in this system.



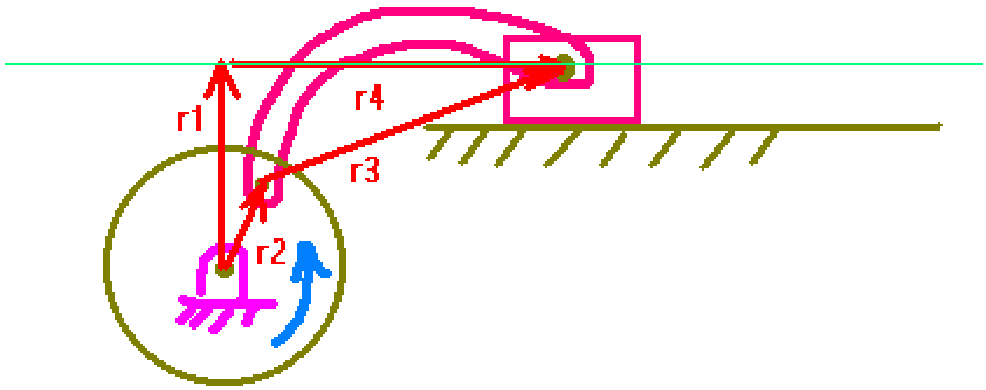


3) Revolute joints:
Revolute joints attach links allowing one degree of freedom motion, and show up in loop closure equations.



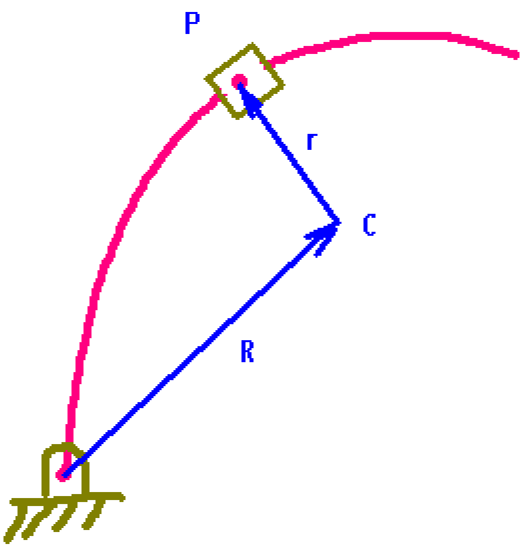
4) Slider elements:

Represent sliders with a vector, running along the axis of the slider, starting from some reference. This vector has a variable length (unknown unless driven) and either fixed or variable angle



5) Sliders on curved beams:

Referring to the fig. below, **C** = center of curvature, **R** is from ground to **C**, and **r** is from **C** to **P**. Note that in an instantaneous sense, **R** is constant relative to the bar, and **r** has constant length. Thus, the only unknown in referencing **P** is the angle of **r**.



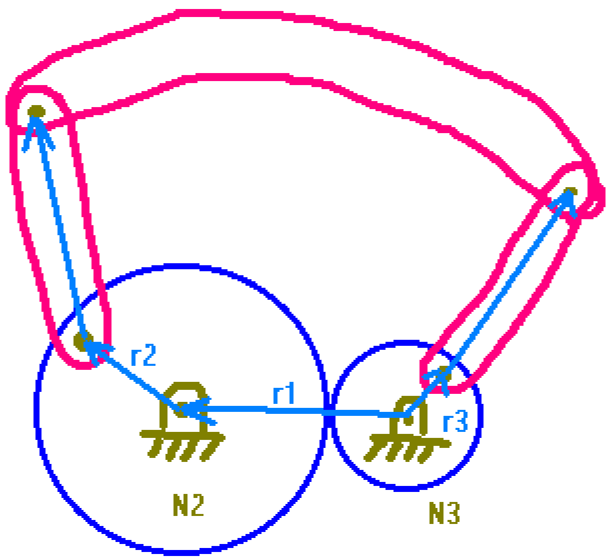
6) Hydraulic cylinders

Here, **r** has variable length and angle. Note that since hydraulic cylinders are often input devices, typically the length of **r** is known, and the angle is unknown.

7) Gears:

No vectors are drawn to the point of contact, rather b/n the centers of the gears. The contact, a 1 dof joint, is described with a scalar equation relating the two gear vectors:

$$\theta_3 = -{}^{N_2}\!/_{N_3}\,\theta_2 + \alpha_{offset}$$



8) Belts and pulleys, Chains and Sprockets:

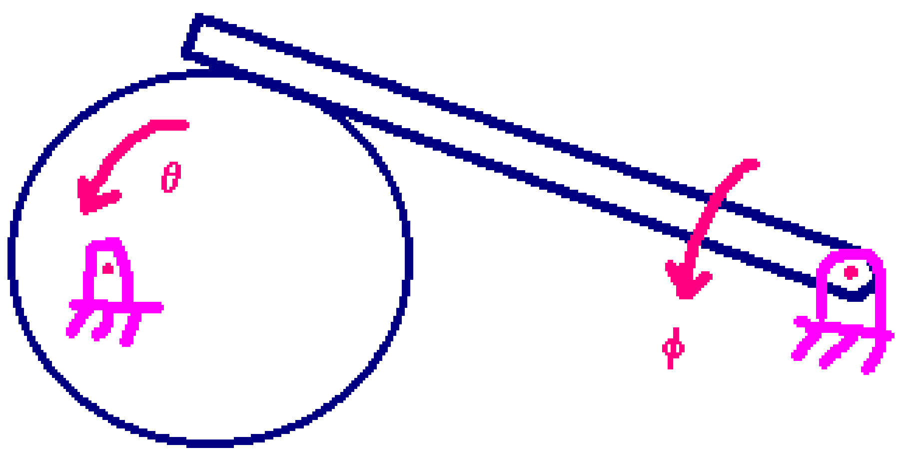
Model in the case of gears using the constraint equation:

$$\theta_3 = -\frac{D_2}{D_3}\theta_2 + \alpha_{offset}$$

9) Cam Systems:

Use the displacement function, *f* to relate the follower to cam motion:

$$s = f(\theta), \quad \phi = f(\theta)$$



Procedure (cont.):

Step 1: Draw a schematic of the mechanism.

- This will clear identify the links and joints.

Step 2: Perform Mobility analysis.

- Determine the number of dof and input actuators.

Step 3: Create a vector model of the mechanism.

- By replacing all rigid bodies in the mechanism with appropriate, representative vectors.
- Note; kinematically, a link is defined as a rigid, straight-line connection between two joints.

Step 4: List the unknowns in the model

- The unknowns in the analysis model consist of all the variable parameters within the model, minus the number of inputs (dof)
- Justification: It is assumed that any fixed parameter in the model can be measured at any time, and will be known from that point forward.

Step 5: Write the position equations needed to solve.

- There are 2 types of equations: loop closure equations (usually vector), and constraint equations (usually scalar).
- Be careful that loop equations written are unique.

$$LE = \frac{VAR - M - CE}{2}$$

Step 6: Solve the equations

Notes:

i. This is a system of *n* nonlinear equations with *n* unknowns (and generally coupled in the unknowns). Therefore, numerical solution techniques must be used to solve the in the most general case.

ii. Closed form solutions exist if the equations can be decoupled to sets of 2 eq's with 2 unknowns.

iii. Closed-form solutions are more common

iv. Solution techniques for solving loop equations will follow

Example 1: Mechanism Modeling
Given the 3-point hitch device on a tractor, create a mechanism model that will solve for the location of the output pivots as a function of the inputs.

Example 1 (cont.)
Step 1: Draw a Schematic
Step 2: determine mobility
Step 3: Create vector model
Step 4: List the unknowns:
Step 5: Write Equations
Step 6: Solve (next topic)

Example 2: Mechanism Modeling

<u>Example 1 (cont.)</u>

Step 1: Draw a Schematic
Step 2: determine mobility
Step 3: Create vector model
Step 4: List the unknowns:
Step 5: Write Equations
Step 6: Solve (next topic)

Mechanism Examples

Practice Mechanism Modeling on the following Examples:

Front-End loader

Skid Steer

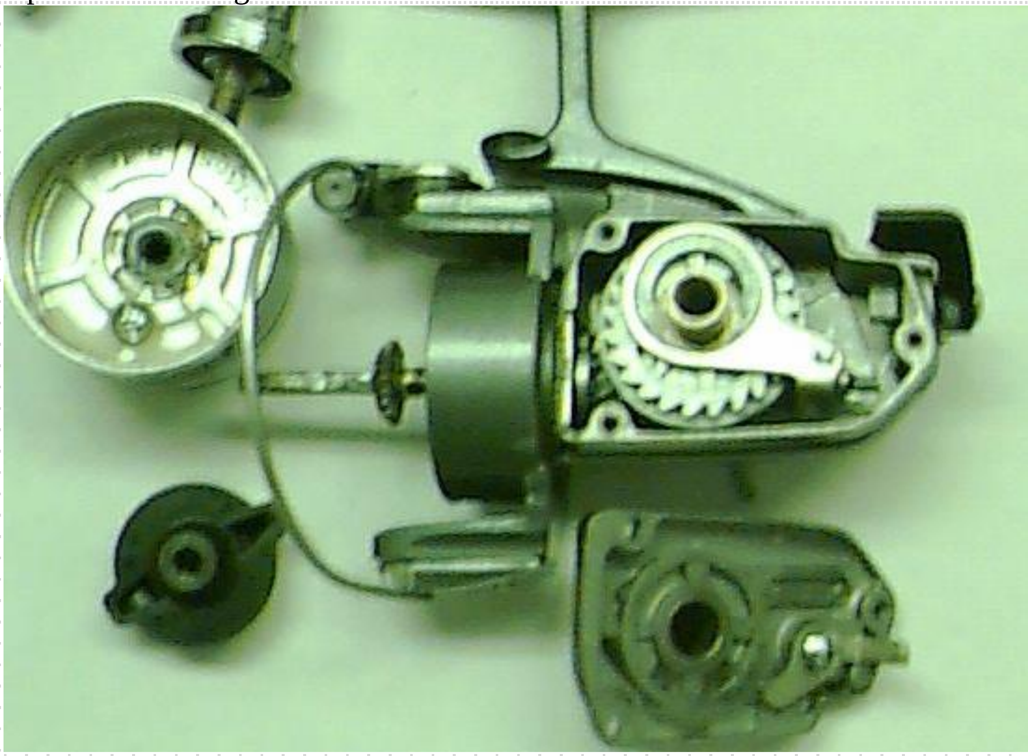
Leg Press Machine

Double A-Arm Suspension

Backhoe

Air Pump

Open-faced fishing reel



Roller-blade brake

Vice-grips

Bike Suspension

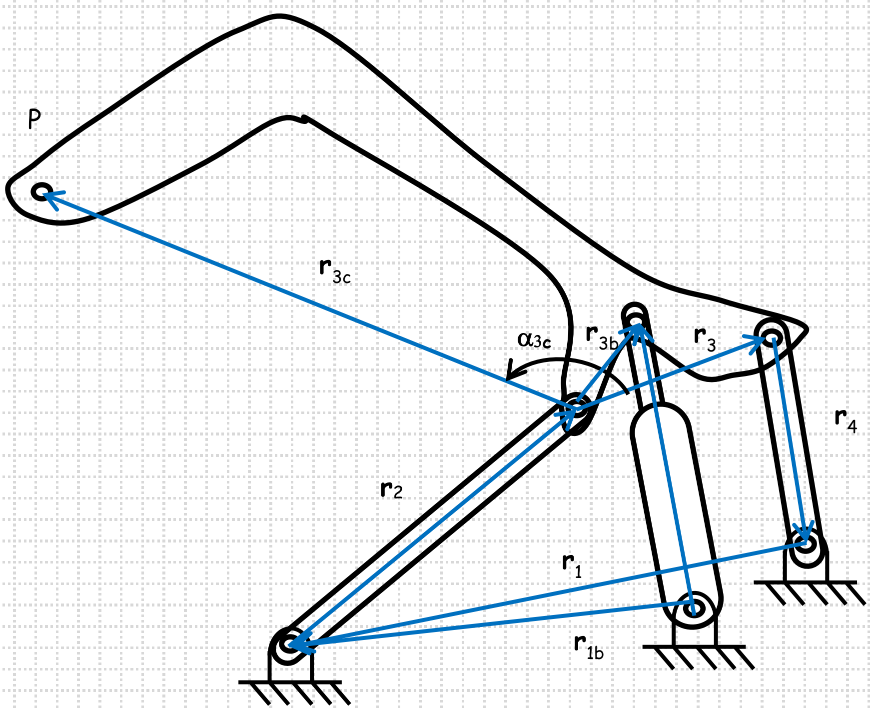
Leg curl machine

Bobcat 650S loader: http://www.youtube.com/watch?v=aqK0qsXH3e4

Bobcat S650 Skid Steer Loader

The new Bobcat® S650 skid steer loader is part of the M-Series. With its vertical lift path, the S650 gives you increase lifting capacity and provides greater forward reach at full lift height for loading over dump trucks.

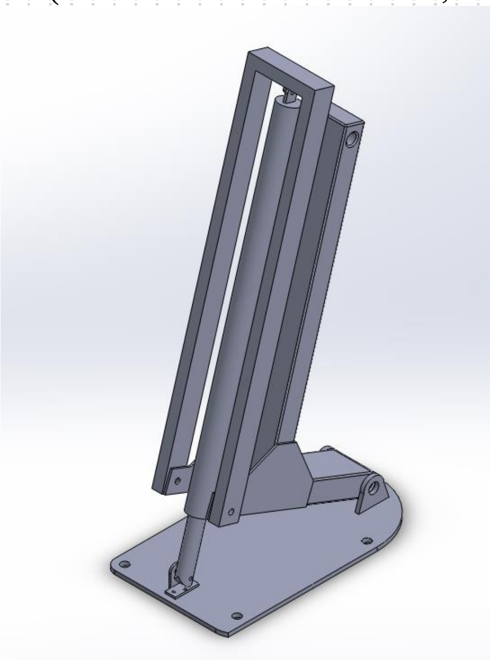

601
Bobcat

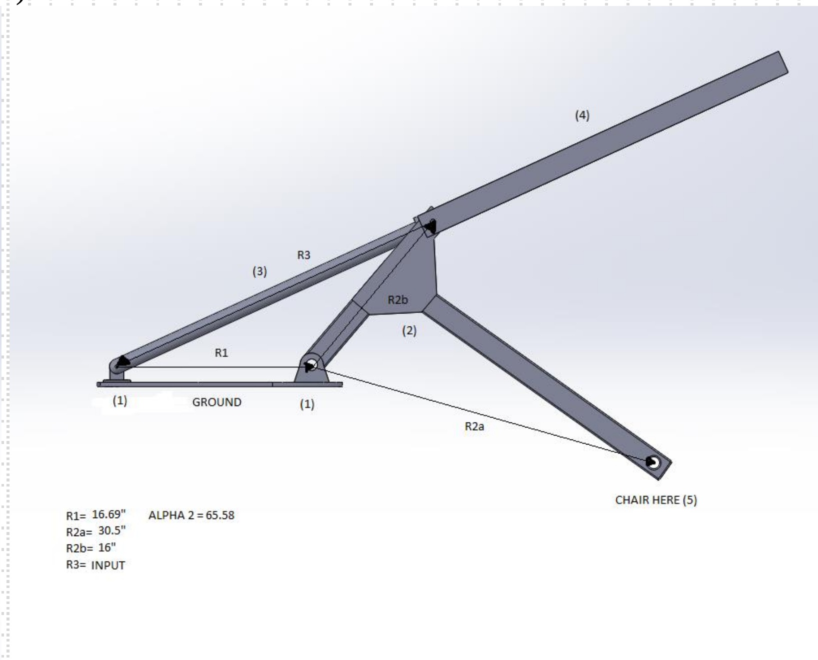

P
$\mathbf{r}_{3c}$
α_{3c}
$\mathbf{r}_{3b}$
$\mathbf{r}_3$
$\mathbf{r}_4$
$\mathbf{r}_2$
$\mathbf{r}_1$
$\mathbf{r}_{1b}$

1. Schematic (see above)
2. Mobility: $M = 3(6-1) - 2(7) - 0 = 1$
3. Vector model (see above)
4. unknowns $\theta_2, \theta_3, \theta_{3b}, \theta_{3c}, \theta_4, \theta_5;\ r_5 = input$
5. Equations

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 = 0$$
$$\vec{r}_2 + \vec{r}_{3b} - \vec{r}_5 + \vec{r}_{1b} = 0$$
$$\theta_{3b} = \theta_{3c} + \alpha_b, \theta_{3c} = \theta_{3b} + \alpha_c$$

Pool Lift
(Taken from the Pool-lift team, F12)





Floating Arm Trebuchet: