Part II: Kinematics, the fundamentals

This section will provide an overview of the fundamental concepts in kinematics. This will include the following topics:

1. What is Kinematics?
2. Kinematics within mechanics
3. Key definitions
4. Motion and kinematic pairs
5. Transmission of motion
6. Mobility
7. Review of some general classes of Mechanisms

1: What is Kinematics?

"Kinematics is the study of ____

Kinetics is the study of _________ on systems in motion:

Underlying Assumptions in Kinematics:

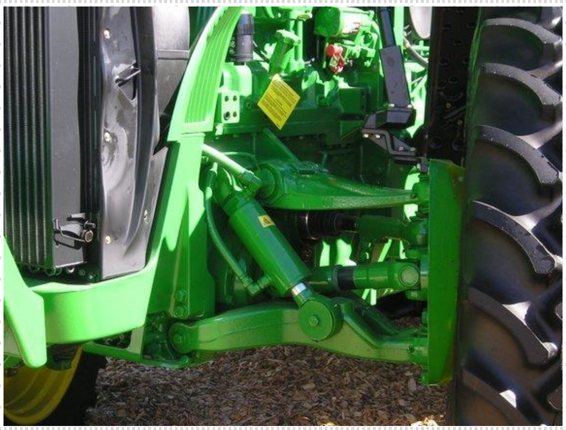
1) Rigid bodies

2) Ignore forces (applied, friction, etc)

3) Bodies are connected by Joints – Kinematic pairs

4) The nature of connection b/n kinematic pairs is maintained

This is kinematics,



and this is kinematics,

This is NOT kinematics,

Why Kinematics? Why Machines?

Create / harness energy (non-human energy)

Manufacturing, agriculture

Assistive / serve humans

What is the future of kinematics and machinery?

Miniature, micro and perhaps even nano-scale machines (motion at a micro or molecular level)

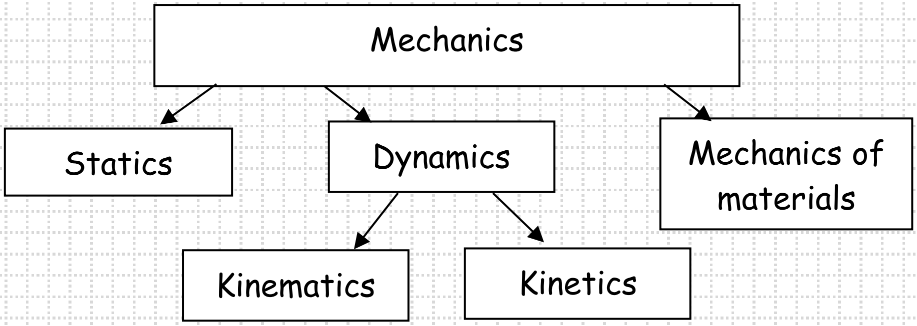
Medical, rehabilitative, prosthetic

Self-replication of machinery, machines design machines

Machines become more biological in nature (compliant, biological muscles, intelligent).

2. Kinematics within Mechanics

Kinematics and the theory behind machines have a long history. Kinematics has evolved to become a unique component within Mechanics as demonstrated in this figure



3. A few key definitions:

Kinematics

Dynamics

Kinematic Chain

Mechanism

Machine

Crank

Rocker

Coupler

Degrees of freedom

Constraint

Mobility

4. Motion (of a rigid body): displacement of a rigid body w.r.t. a fixed frame or reference frame (for dynamics, needs to be an IRF).

Translation:

Rotation:

Planar:

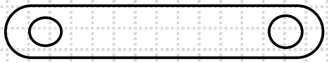
Spatial:

Kinematic Pairs: Two members (links) are jointed through a connection (joint) that defines the relative motion b/n the two.

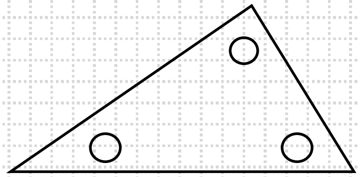
Links

Joints

Some examples of Links include



Binary Link



Ternary Link

Quaternary Link 1

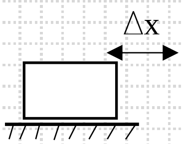
Classification of Joints

Joints are classified based on the following

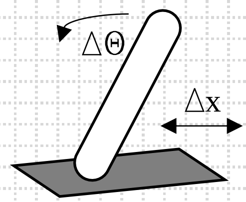
- **A.** Number of degrees of freedom allowed at the joint
- **B.** Number of links joined
- **C.** Type of contact between the elements i.e line contact, point contact or surface contact
- **D.** Type of physical closure of the joint

A. Number of degree of freedom allowed at the joint

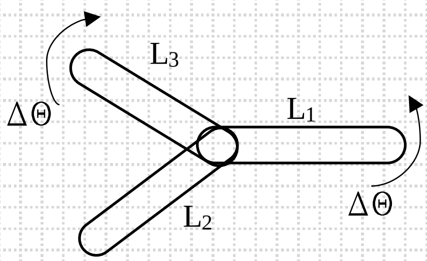
- Prismatic Joint : One degree of freedom



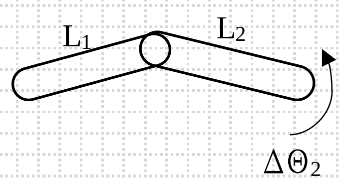
- Link against plane : Two degrees of freedom



B. Number of Links Joined



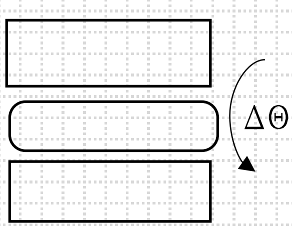
Second order pin Joint- two DOF 2



First order pin Joint- One DOF 1

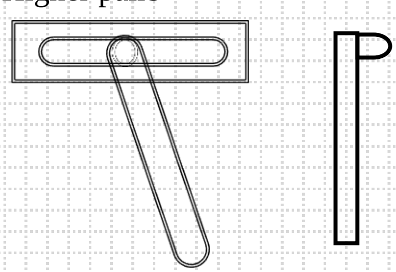
Types of contact

- Lower Pair



Revolute Joint

- Higher pairs



Pin in Slot

More about joint

Classically classified into a couple classes: Higher pair (point contact) and Lower pair (line contact).

Various types of joints:

Revolute:

Prismatic (slider)

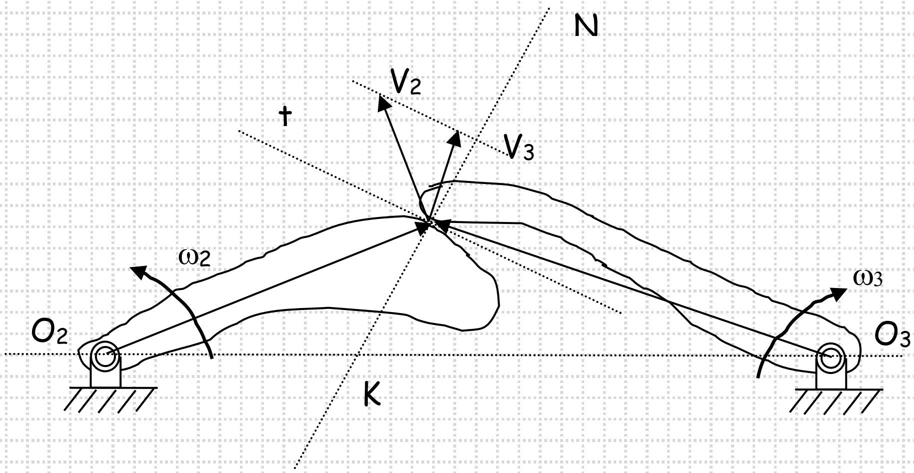
Cam or gear

Rolling contact

Spring

Others?

5. Transmission of Motion: The motion of a mechanism is defined by its constraints (kinematic). The following example shows one of the most general cases of motion between two bodies and demonstrates some key elements in understanding the behavior of motion. Consider two general kinematic bodies (rigid bodies, known geometric properties) in contact at point **P**. Each body rotates about a fixed point, O2 and O3.



Notes:

1) A common Normal and tangent (N, t) exist and are defined by the 2 surfaces
2) "Condition of contact": no relative motion can occur along the common normal
3) All sliding takes place along the common tangent
4) The result of these rules (plus some geometric construction):

$\omega_2 = {}^{V_2}/_{O_2P} \quad \omega_3 = {}^{V_3}/_{O_3P}$

$\frac{\omega_3}{\omega_2} = \frac{O_2K}{O_3K}$

5) Requirement for constant velocity:

6) Requirement for no sliding:

5. Kinematic schematics, Diagrams:

A kinematic schematic or diagram is drawing that clearly defines the kinematic nature of a mechanism. In particular, these drawings must clearly show the following:

1) Each body, the number of bodies, and the type of body (binary, ternary, etc).
2) Each joint or kinematic pair, and its type.
3) Which bodies are coupled with kinematic pairs.

For naming purposes in Norton:

<u>Kinematic Schematic;</u>

<u>Kinematic diagram:</u>

<u>Example:</u>

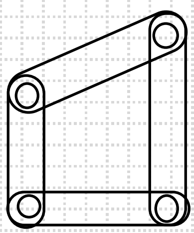
6. Mobility Analysis:
Mobility is defined as the number of dof. Mobility is calculated as the total number of possible degrees of freedom, minus the number of constraints. The following diagrams will demonstrate the process:

Item	Diagram	DOF
One body		
Two Bodies		
Two bodies connected by a revolute		
Ground (it is a body)		

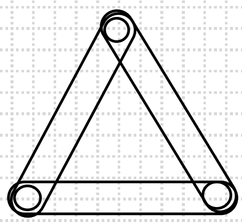
Writing these rules as equations yields:

Which is known as Grubler's or the Kutzbach equation.

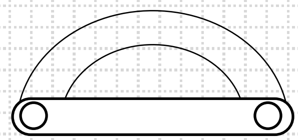
Note: when M = 0 $\rightarrow$ Structure, statically determinant
M < 0 $\rightarrow$ Indeterminant structure
M > 0 $\rightarrow$ Mechanism with M dof



Mechanism DOF = 1



Statically determinate DOF = 0



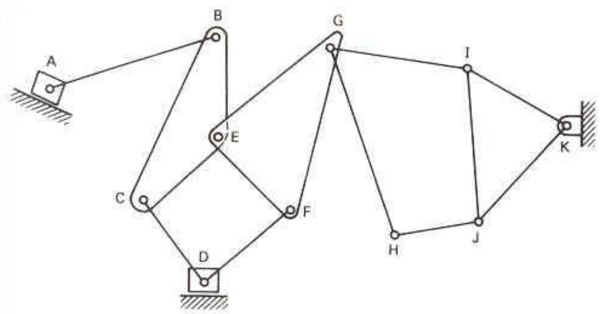
Statically indeterminate DOF = -1

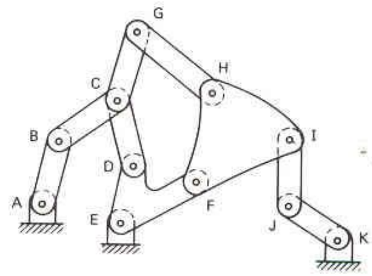
Number Synthesis:
This is the determination of the number and the order of links and joints necessary to produce motion of a particular degree of freedom

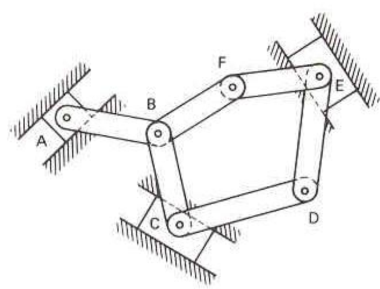
Example of a 1 DOF planar mechanism with Revolute Joints and up to 8 links

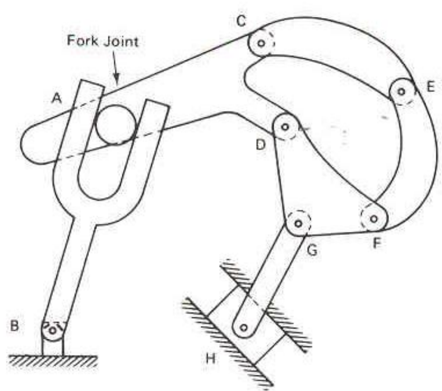
Total Links	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

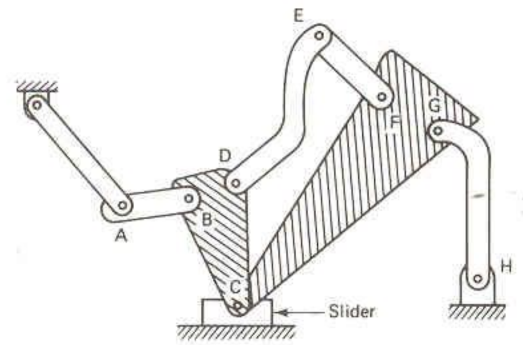
Mobility Examples (list 4-6 examples for in-class practice 3 line drawings, 3 photos)

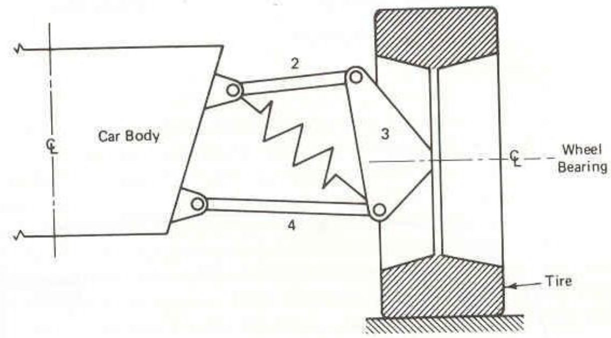


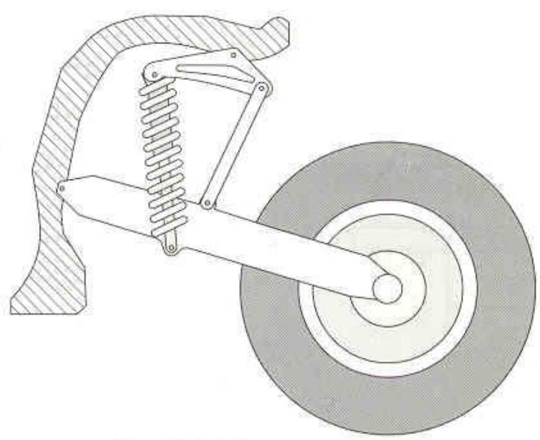












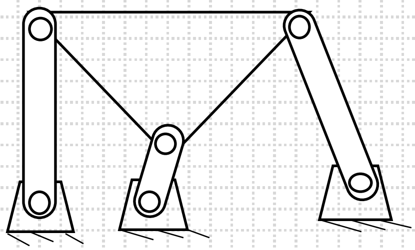
Suzuki Full-floater Suspension

Figure P1.96

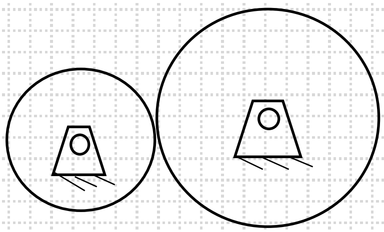

CASE
Lx162

Paradoxes

This is a case whereby the Gruebler criterion gives a wrong/ misleading DOF. This occurs due to the fact that the Gruebler's equation does not pay attention to the geometry of the object. An example of link arrangement that exhibits such paradox is the E-Quintet.



The E-Quintet with DOF =0 agrees with Gruebler equation



Rolling cylinders- DOF =1 Gruebler predicts zero. This is a full joint , pure rolling no slip

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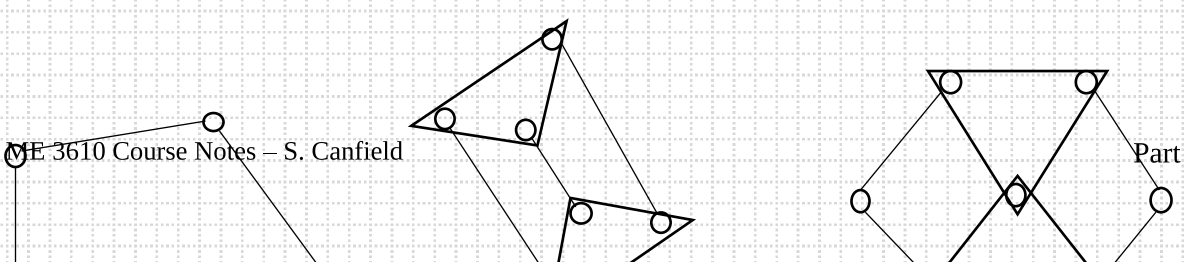
Isomers

This means having equal parts. Links have various nodes available to connect to other link's nodes in different ways. So, the motion properties of a linkage assembly differs due to the connections of available links.
Only one isomer exists for a case of four links. An isomer is unique if the interconnections between its types of links are different. This means that all binary links are considered as equal. A six link case of 4 binaries and 2 ternaries has only two isomers. These are known as the watt's chain and the Stephenson's chain.



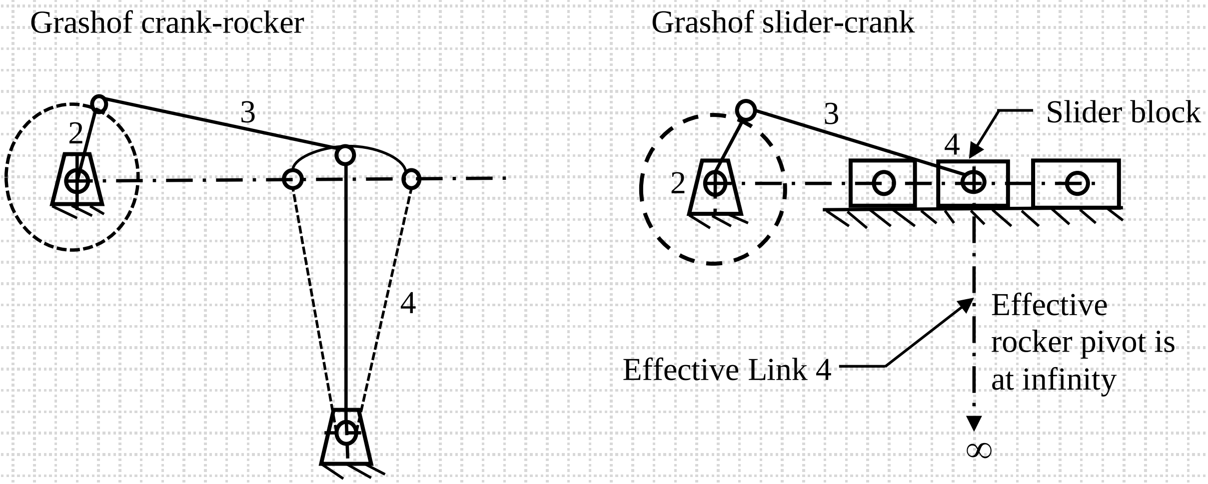
Linkage Transformation

This technique gives the designer a toolkit to basic linkages of a particular DOF. In this technique, the designer is not constrained to using only full, or rotating joints. So, the designer can transform basic linages to a wider variety of mechanisms with greater usefulness.

Several techniques that can be applied are:

1) Any full rotating joints can be replaced by a sliding full joint with no change in DOF of the mechanism
2) Any full joint can be replaced by a half joint, but this will increase the DOF by one
3) Removal of a link will reduce the DOF by one
4) The combination of items 2 and 3 above will keep the original DOF unchanged
5) Any ternary or higher-order link can be partially "shrunk" to a lower-order link by coalescing nodes. This will create a multiple joint but will not change the DOF of the mechanism
6) Complete shrinkage of a higher-order link is equivalent to its removal. A multiple joint will be created, and the DOF will be reduced.

Examples



Transforming a fourbar crank- rocker to a slider- crank

In this example, a fourbar crank-rocker linkage had been transformed into a fourbar slider-crank by the application of rule #1. It is still a fourbar linkage. Link 4 has become a sliding block. The Gruebler's equation is unchanged at one degree of freedom because the slider block provides a full joint against link1 (ground), as did the pin joint it replaces. Note that this transformation from a rocking output link to a slider output link is equivalent to increasing the length (radius) of rocker link 4 until its arc motion at the joint between links 3 and 4 becomes a straight line. Thus the slider block is equivalent to an infinitely long rocker link 4, which is pivoted at infinity along a line perpendicular to the slider axis as shown in the figure above.

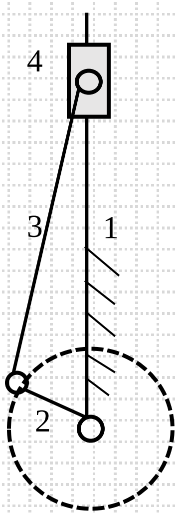
Intermittent Motion

This is a sequence of motions of dwells. A dwell is a period in which the output link remains stationary while the input link continues to move. There are many applications in machinery which require intermittent motion. Some examples of mechanism that exhibits such motions are:

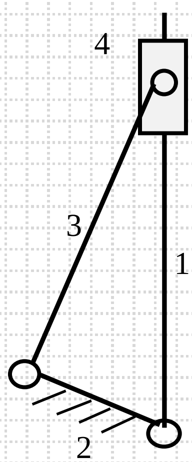
1) Geneva Mechanism
2) Ratchet and Pawl
3) Linear Geneva Mechanism

Inversion

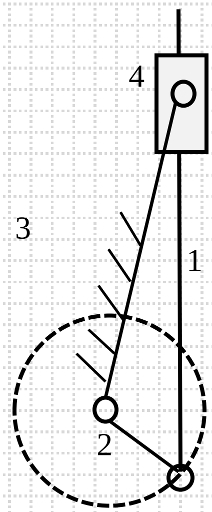
This is created by grounding a different link in the kinematic chain. There are many inversions of a given linkage as has links and the motions resulting from each inversion can be quite different. However, some inversions of a linkage may yield motions similar to other inversions of the same linkages. In these cases, only some of the inversions may have distinctly different motions. Those inversion which have distinctly different motions are denoted as **Distinct Inversions**.



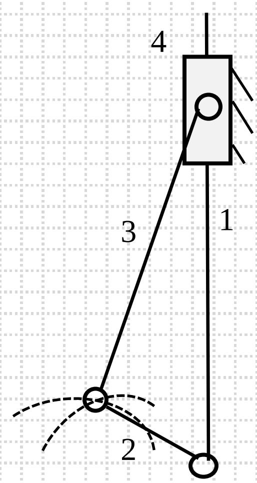
A) Inversion #1 Slider block translates



B) Inversion #2 Slider block has complex motion



C) Inversion #3 Slider block rotates



D) Inversion #4 Slider block is stationary

Other linkage assembly inversion include:
The Watt's sixbar chain: This has two distinct inversion
The Stephenson's sixbar chain: This has three distinct inversion

The Grashof Condition

This is a very simple relationship which predicts the behavior of a fourbar linkage's inversions based only on the link lengths.

Where S= length of the shortest link
L=length of the longest link
P=length of one remaining link
Q=length of the other remaining link

If: S+L is less than or equal to P+Q, the linkage is Grashof and at least one link will be capable of making a full revolution with respect to the ground plane. If S+L is not less than or equal to P+Q, then the statement above is not true. This condition is independent of the order of assembly. The motions possible from a four bar linkage will depend on both the Grashof condition and the inversion chosen. The inversion will be defined with respect to the shortest link. The motions are

Case 1 $S+L < P+Q$

In this condition, any link adjacent to the shortest link can be grounded and you get a crank-rocker in which the shortest will fully rotate and the other link pivoted to the ground will oscillate

Case 2 $S+L > P+Q$

In this condition, all inversions will be double-rockers in which no link can fully rotate.

Case 3 $S+L = P+Q$

This is a special case. All inversion will be either double-crank or crank-rockers but will have "change point" twice per revolution of the input crank, when the links all become collinear.