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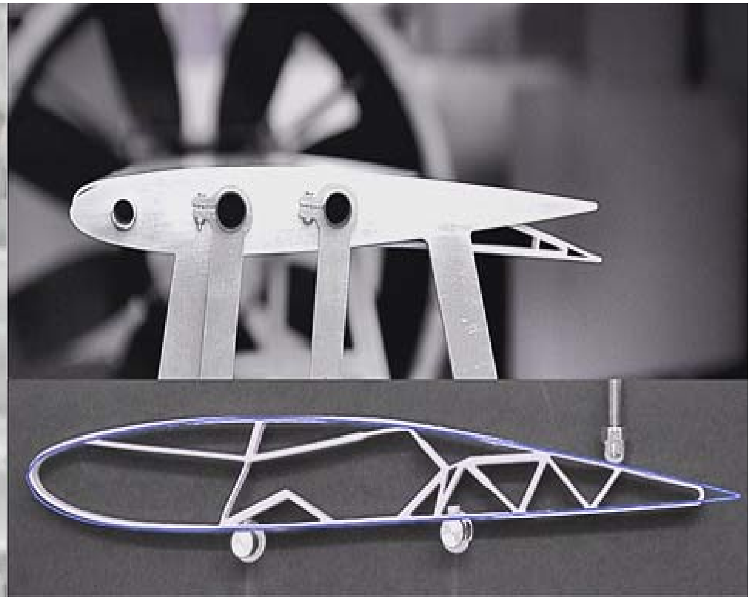
UNIVERSITÄT
ERLANGEN-NÜRNBERG
FACULTY OF ENGINEERING

Engineering Design
Prof. Dr.-Ing. Sandro Wartzack
Prof. Dr. sc. ETHZ Alexander Hasse

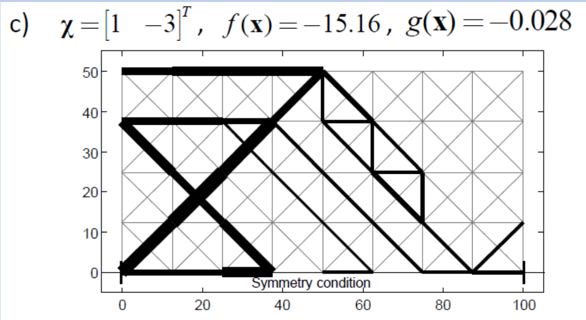
# Synthesis of Compliant Mechanisms via topology optimization

Alexander Hasse

Summer School ASME 2016


# Outline




- Introduction
- What is topology optimization?
- A short introduction to structural optimization
- Topology Optimization of compliant Mechanisms
- Design examples

# Introduction

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**Alexander Hasse**

2001-2007 Diploma Degree in Mechanical Engineering (TU Dresden)

2007-2011 Doctoral Work in Mechanical Engineering (ETH Zurich)

2011-2012 Post-Doc (ETH Zurich)

2012-2014 Head Engineering (Monolitix AG)

Since 2014 Professor for Mechatronic Systems at FAU Erlangen-Nuermberg

# Introduction
## Robotic grippers


Source: Monolitix




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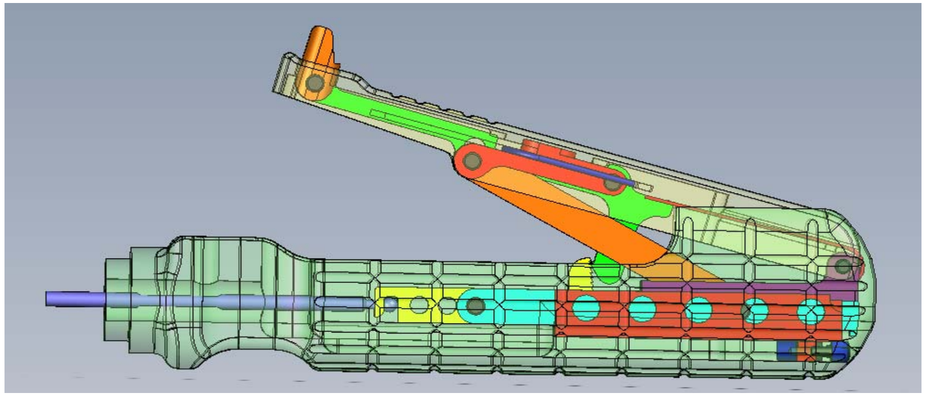
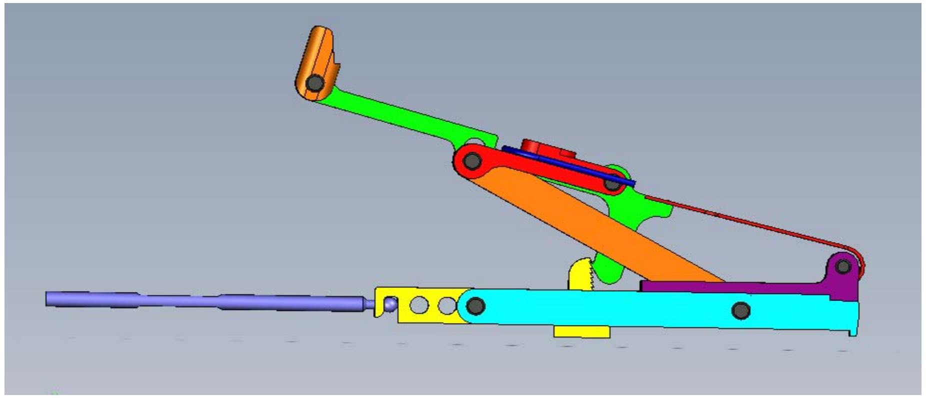
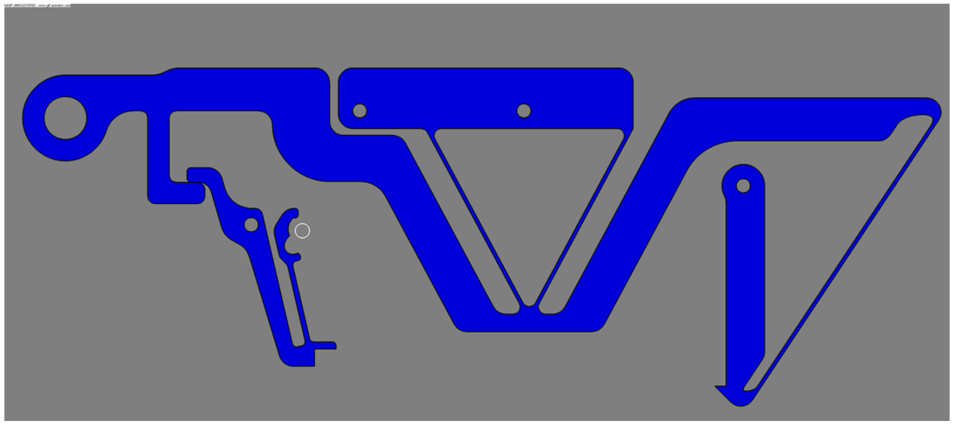
# Introduction
## Medical instruments





**surgical handle – conventional**



**surgical handle – compliant**

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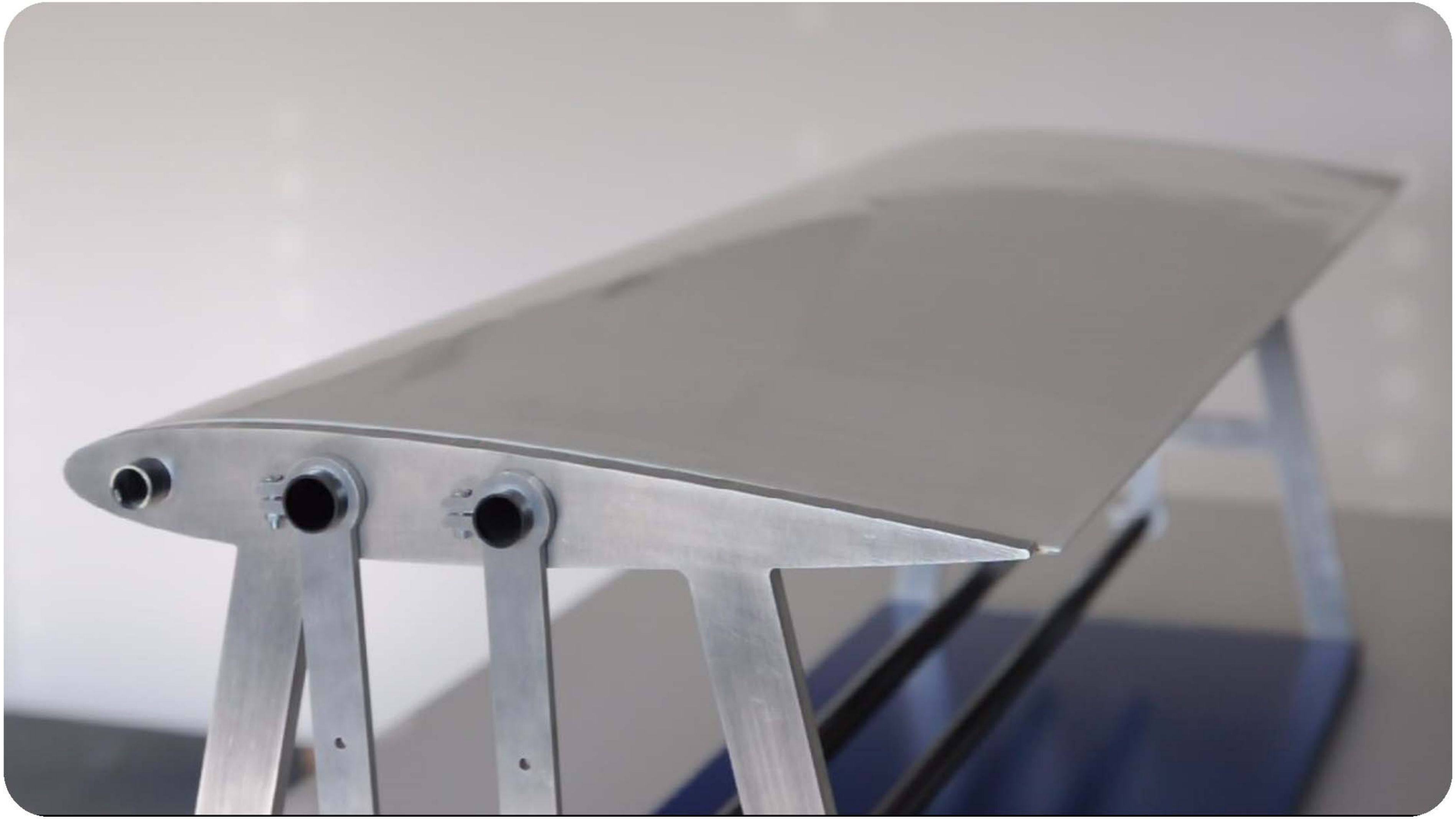
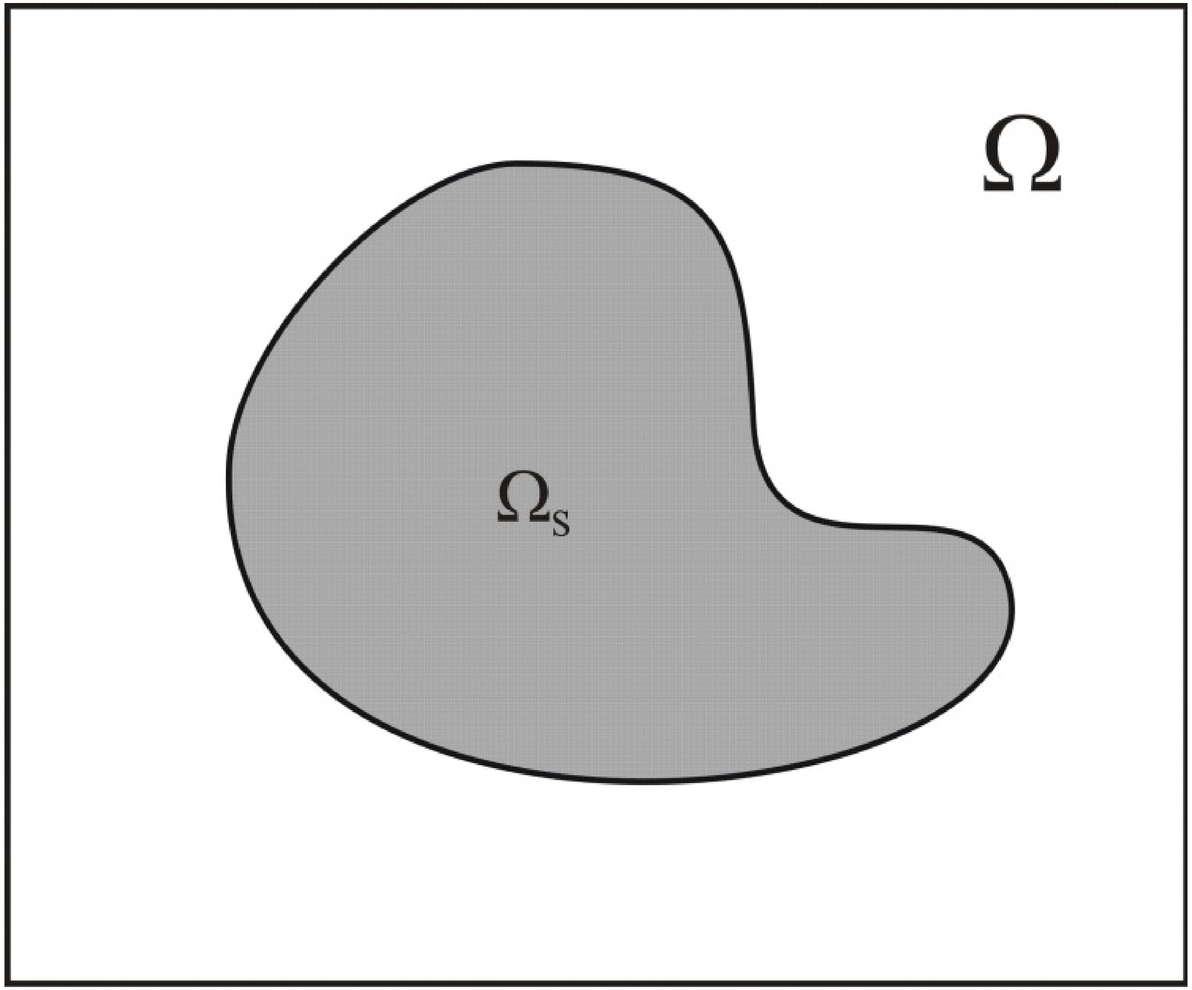
Source: Empa

# Introduction
## Shape adaptive structures



# What is topology optimization?

## The design problem in general







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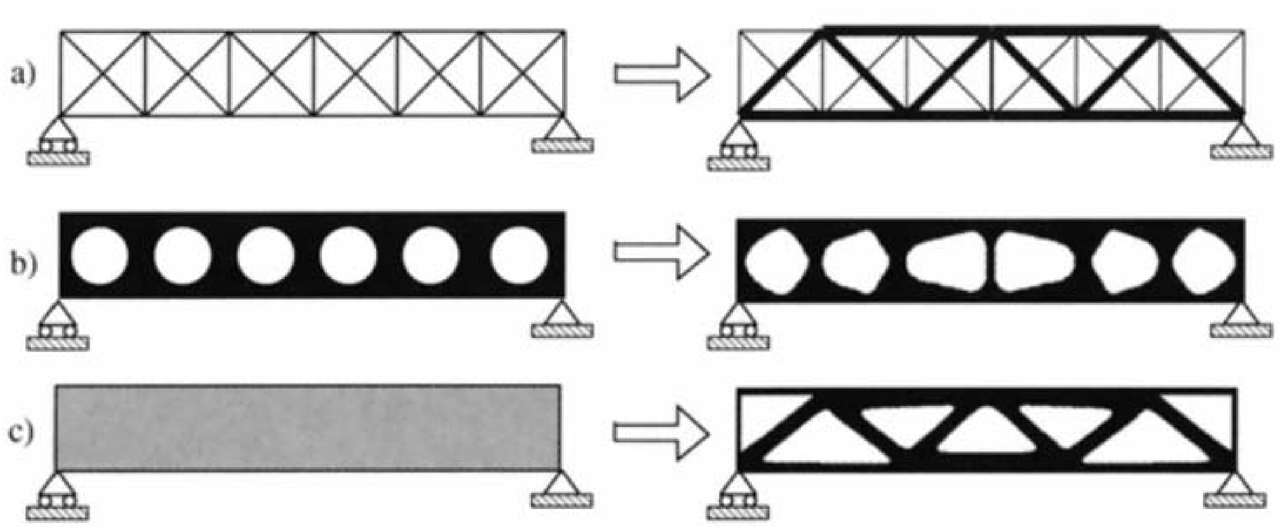
# What is topology optimization?

## Definition of topology, shape and size




Source: Wikipedia, Bendsoe and Sigmund

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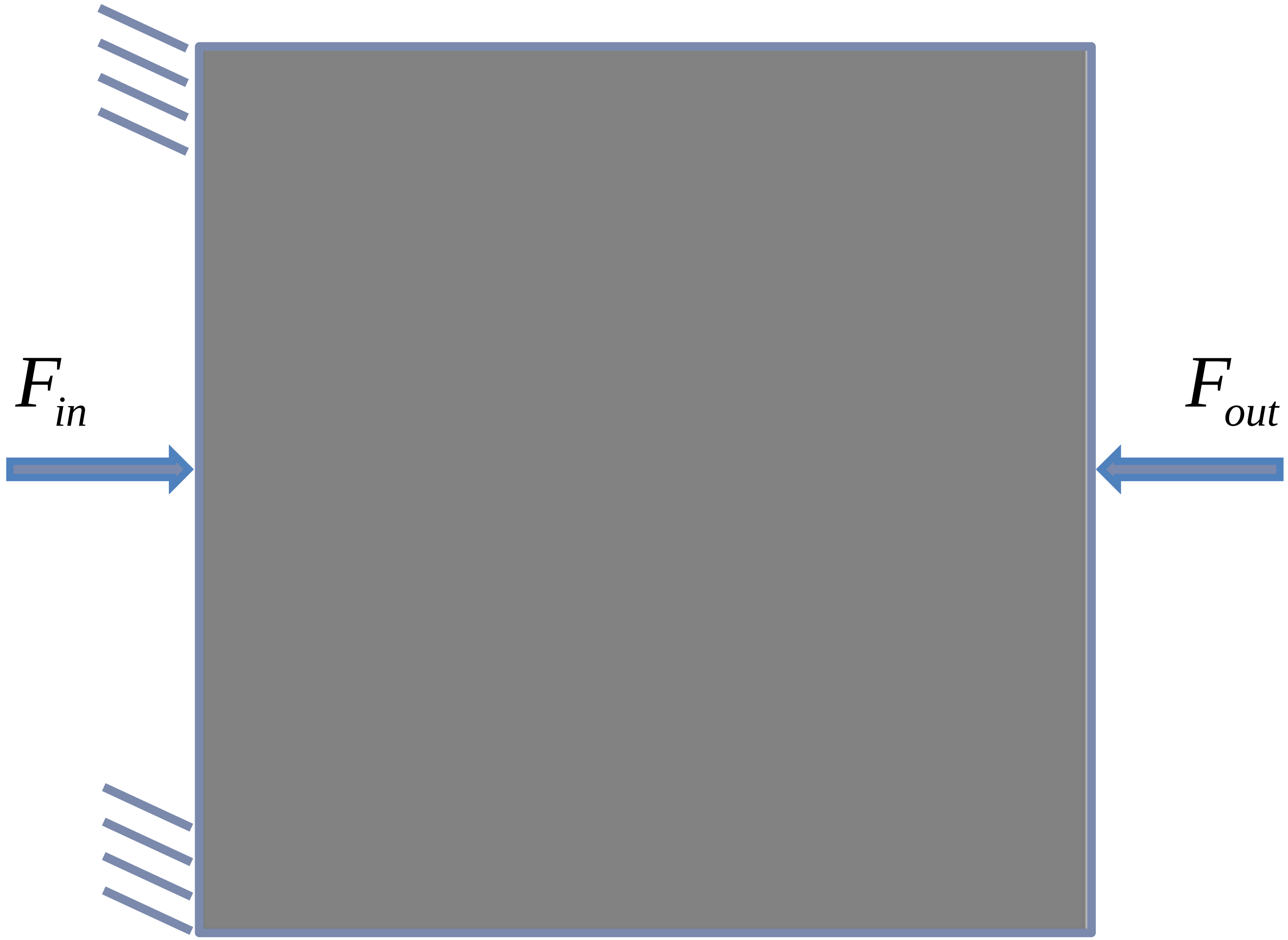
# What is topology optimization?
## Example force inverter







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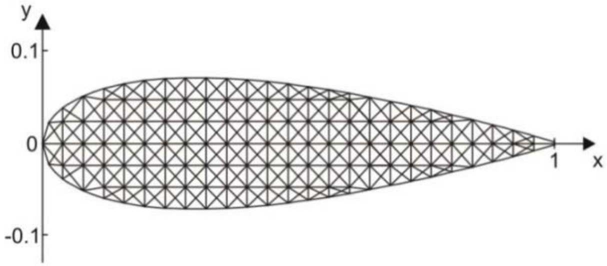
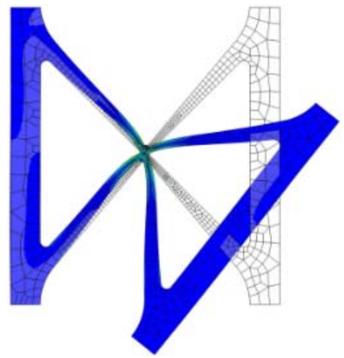
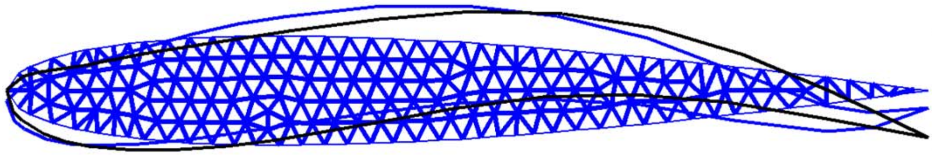
# Structural optimization
## Different modules

Parameterization

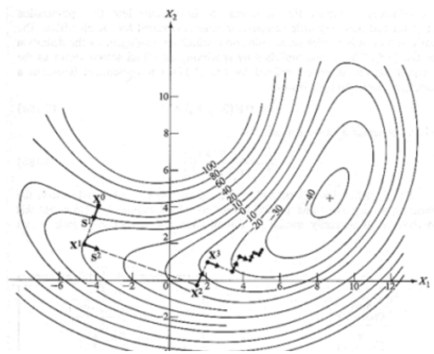



Structural modelling





Optimization algorithm



Opti. formulation

$$\min\ f(\mathbf{x}, \boldsymbol{\varphi}_d)$$

*mit*

$$V(\mathbf{x}) - K = 0$$

Requirements

Source: Kress



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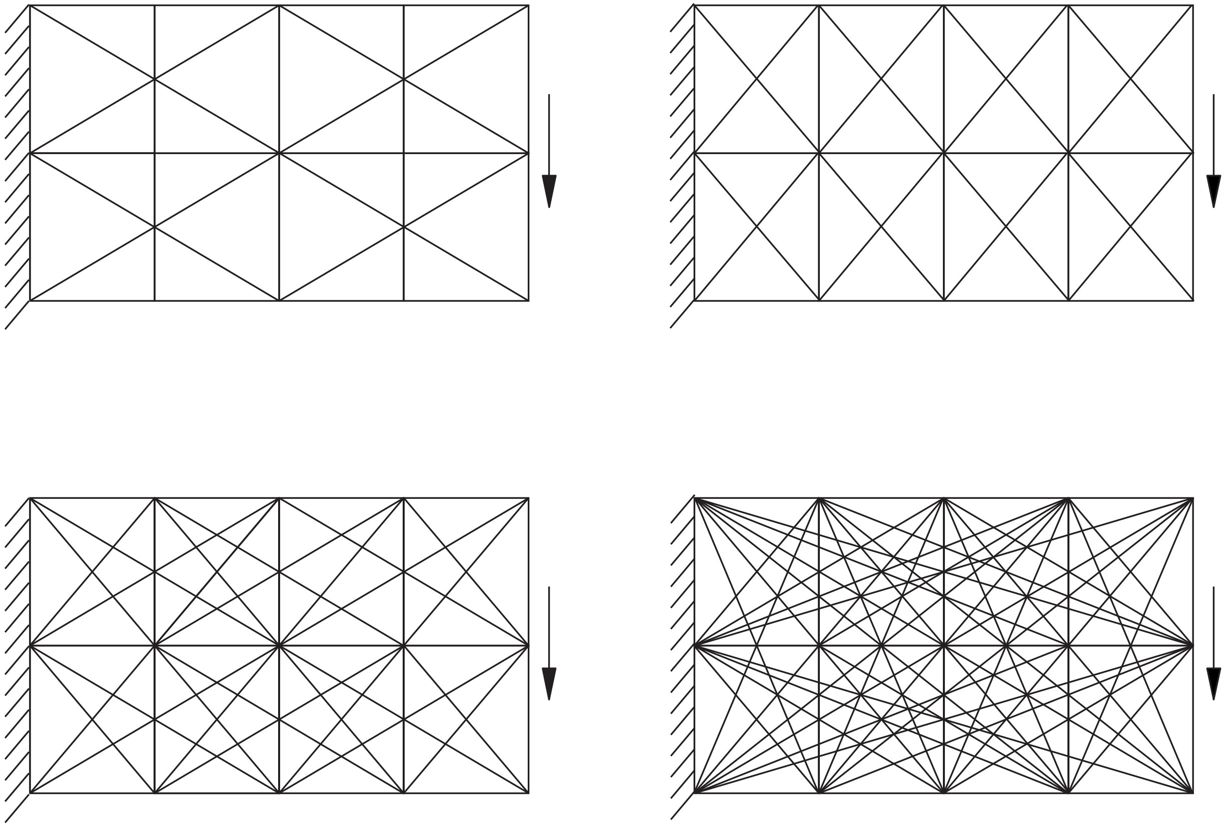
# Structural optimization
## Parameterization and structural modelling

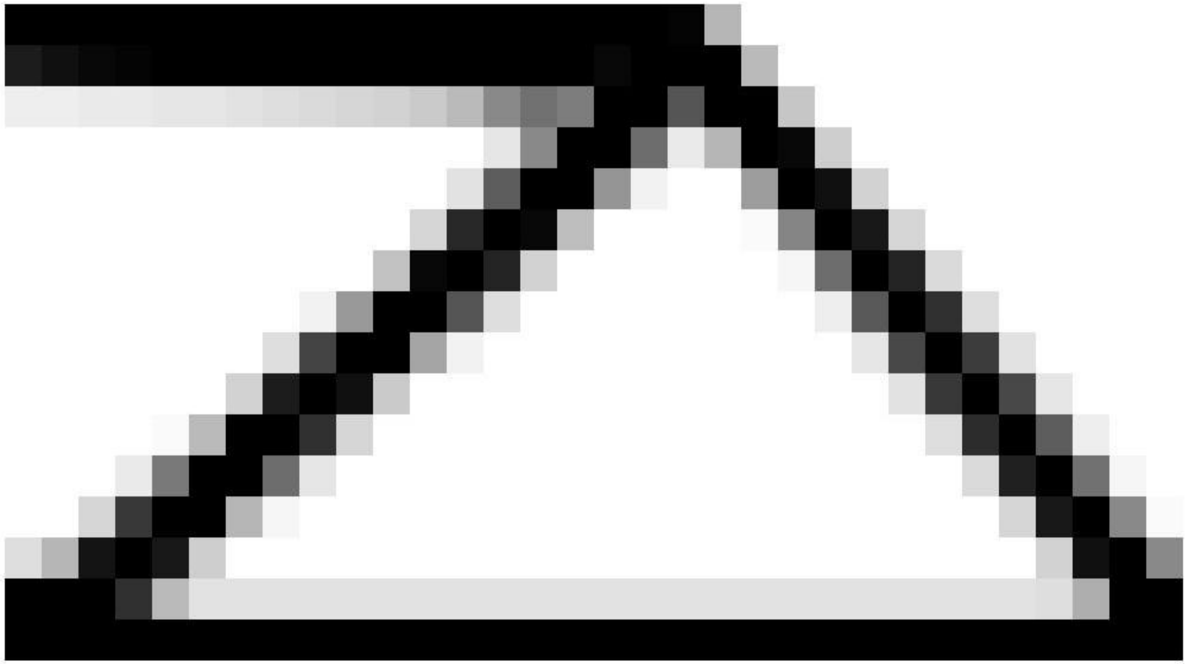
Ground-structure approach



$$\mathbf{k} = \sum_{i=1}^{m} x_i \mathbf{k}^{0(i)}$$

Source: Bendsoe and Sigmund

### Solid Isotropic Material with Penalisation (SIMP)



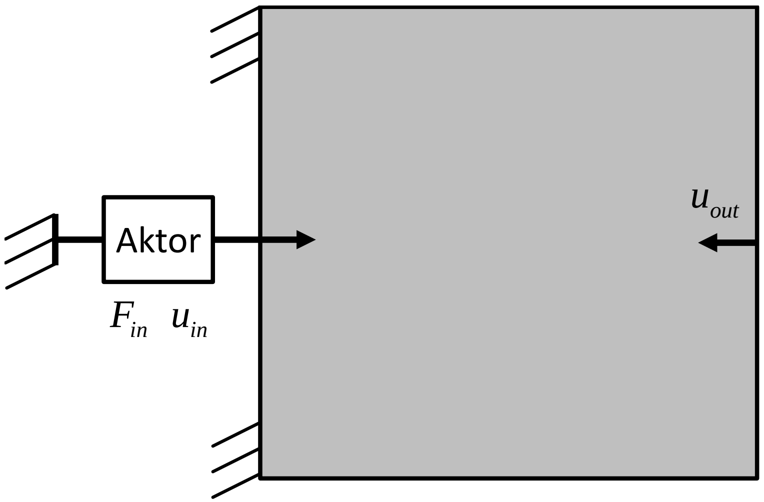
$$\mathbf{k} = \sum_{i=1}^{m} x_i^p \mathbf{k}^{0(i)}$$

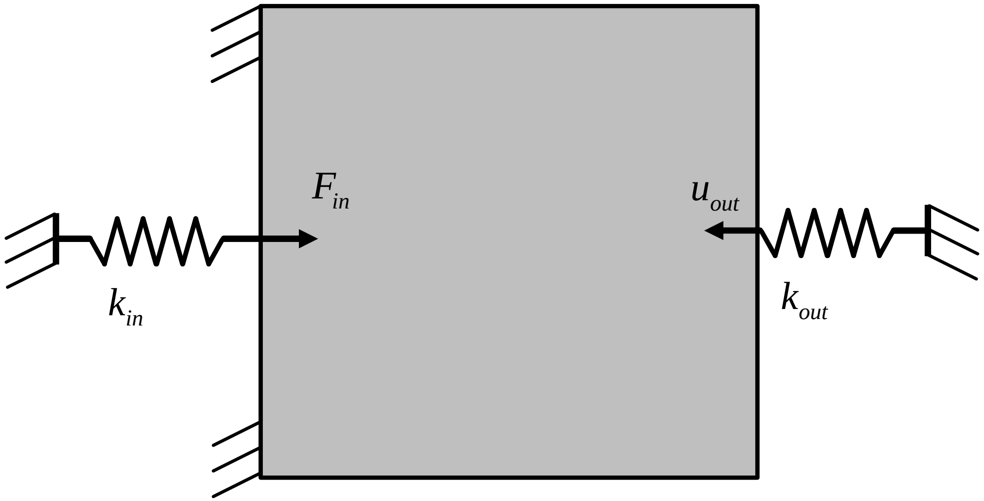
$$m = \sum_{i=1}^{m} x_i v^{0(i)}$$

Source: Bendsoe and Sigmund

# Optimization formulation

## „Spring method“ according to Bendsoe and Sigmund




$$\min_{\mathbf{x}} -u_{out}(\mathbf{x})$$

$$mit$$

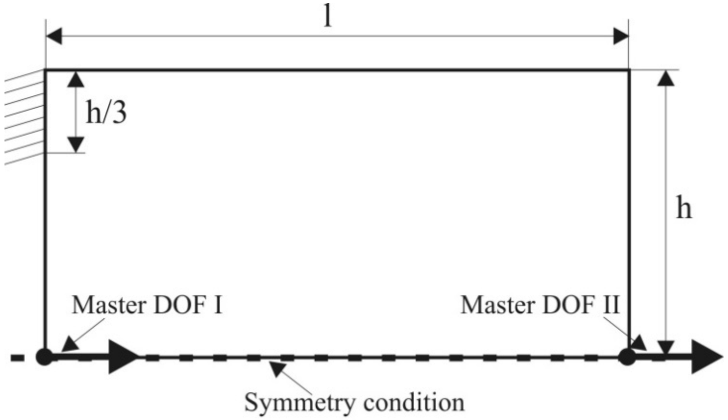
$$\sum_{i=1}^{m} x_i v^{0(i)} \leq V$$

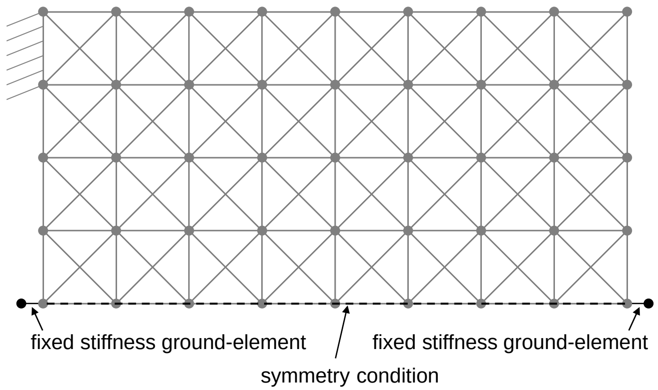
$$0 < x_{\min} \leq x_i \leq 1 \quad , \quad i = 1,\ldots,m$$

# „Spring method“
## Example force inverter

### Preprocessing

1. Build up ground structure




2. Generate „ground“ stiffness matrix for each ground-structure member

$$\mathbf{k}^{0(i)} \quad , \quad i = 1 \ldots m$$



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# „Spring method“

## Example force inverter

### Optimization

Initial design variables

$$\mathbf{x}_0$$

Built stiffness matrix

$$\mathbf{k} = \sum_{i=1}^{m} x_i \mathbf{k}^{0(i)} + \mathbf{k}_{in} + \mathbf{k}_{out}$$

Caculate structural response

$$\mathbf{u} = \mathbf{k}^{-1}\mathbf{f}_{in}$$

$$u_{out}$$

Update design variables by a proper optimization algorithm

$$\mathbf{x}_{new}$$

# „Spring method“
## Example force inverter

### Sensitivity analysis

$u_{out}$ → Update design variables by a proper optimization algorithm → $\mathbf{x}_{new}$

↑

$$\frac{du_{out}}{dx_i}$$

### Calculate sensitivities

$$\frac{du_{out}}{dx_i} = \frac{u_{out}(x_i + \Delta x) - u_{out}(x_i)}{\Delta x}$$

# „Spring method“
## Example force inverter

**Calculate sensitivities analytically**

$$\mathbf{k}\mathbf{u} = \mathbf{f}$$

$$\mathbf{k}\frac{d\mathbf{u}}{dx} = \frac{d\mathbf{f}}{dx} - \frac{d\mathbf{k}}{dx}\mathbf{u}$$

$$\frac{du_{out}}{dx} = \mathbf{z}^T\frac{d\mathbf{u}}{dx} = \mathbf{z}^T\mathbf{k}^{-1}\left(\frac{d\mathbf{f}}{dx} - \frac{d\mathbf{k}}{dx}\mathbf{u}\right)$$




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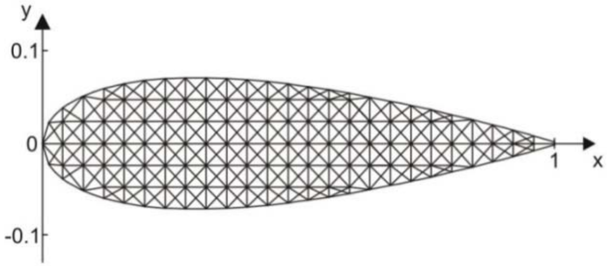
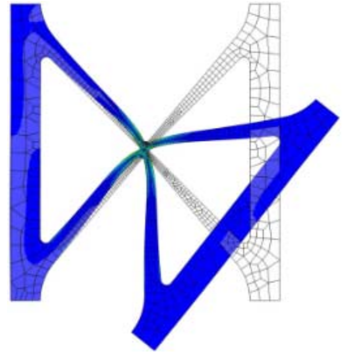
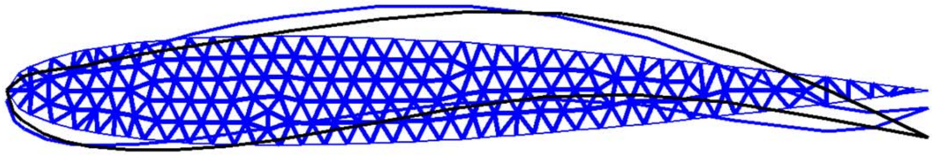
# Structural optimization
## Procedure

### Parameterization




### Structural modelling





Requirements

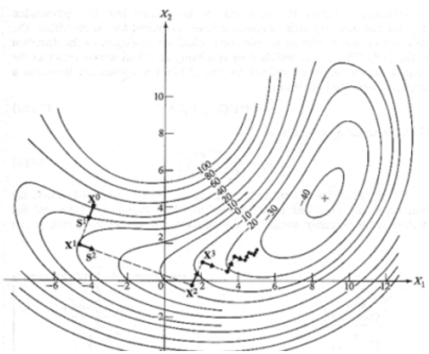
### Opti. formulation

$$\min f(\mathbf{x}, \boldsymbol{\varphi}_d)$$

*mit*

$$V(\mathbf{x}) - K = 0$$

### Optimization algorithm



Source: Kress

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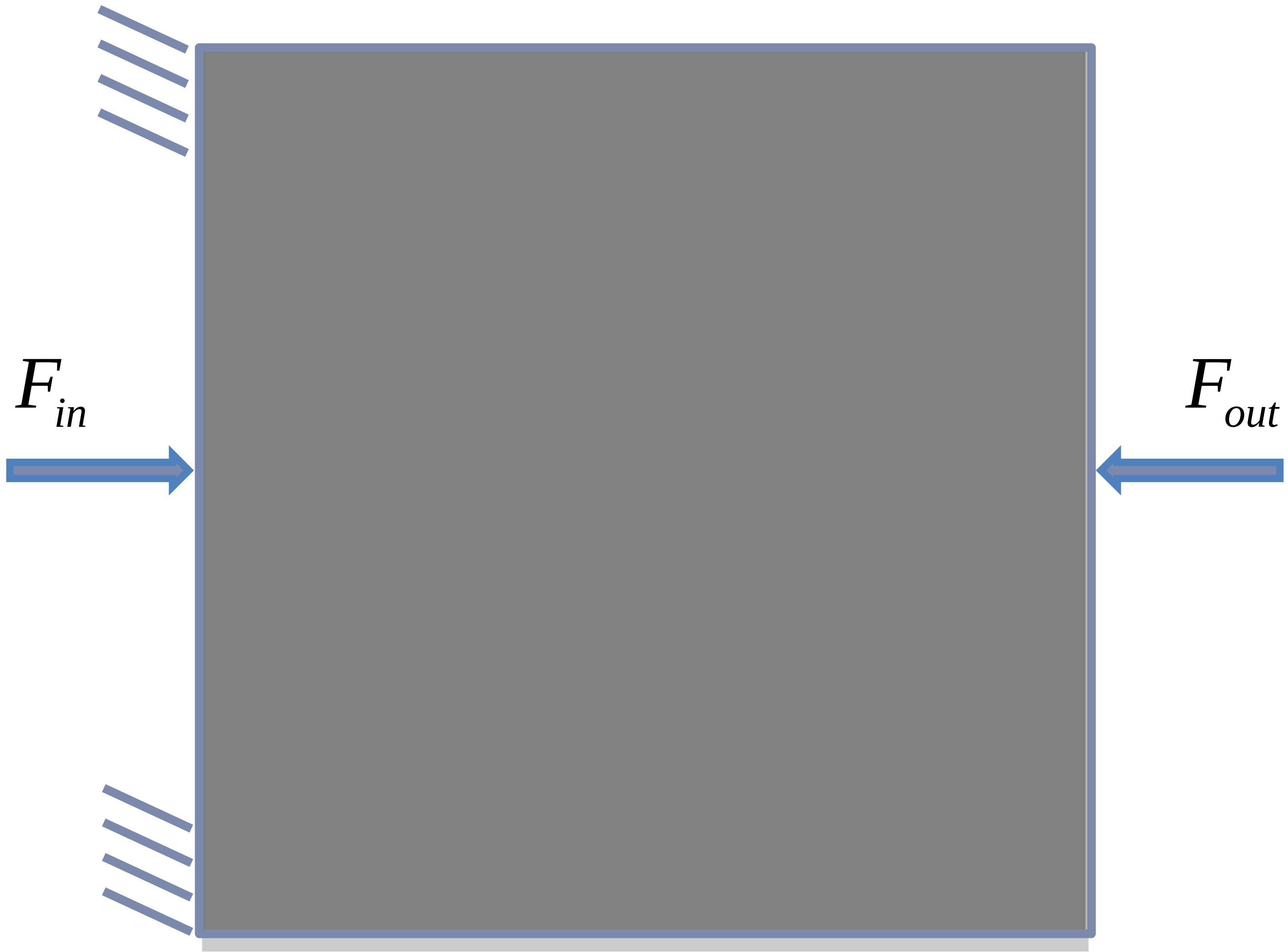
# Structural optimization
## Recommendation




“A 99 line topology optimization code written in Matlab” written by Sigmund with small modifications described in the book „Topology Optimization - Theory, Methods, and Applications”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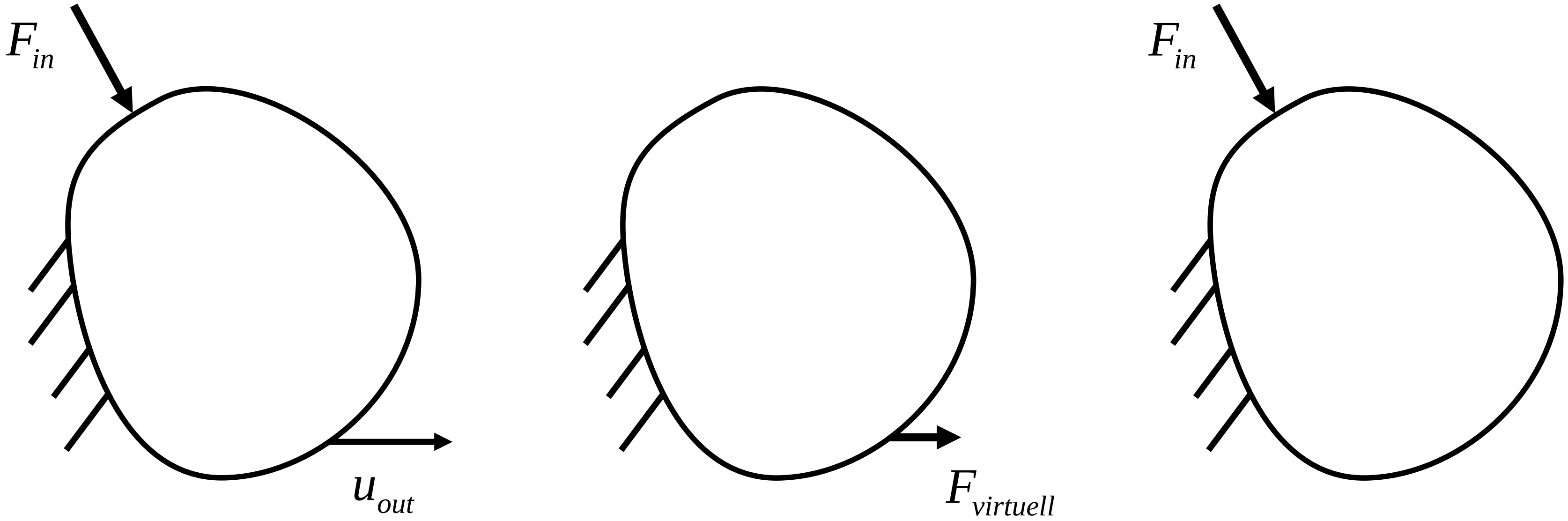
# Optimization formulation

## „MPE/SE“ according to Ananthasuresh




(a) (b) (c)

$$MPE = \mathbf{v}^T \mathbf{k} \mathbf{u}$$

$$\mathbf{k}\mathbf{v} = \mathbf{f}_{virtuell}$$

$$\mathbf{k}\mathbf{u} = \mathbf{f}_{in}$$

$$SE = \mathbf{u}^T \tilde{\mathbf{k}} \mathbf{u}$$

$$\tilde{\mathbf{k}}\mathbf{u} = \mathbf{f}_{in}$$

$$\min f = \frac{-MPE}{SE}$$

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# Optimization formulation

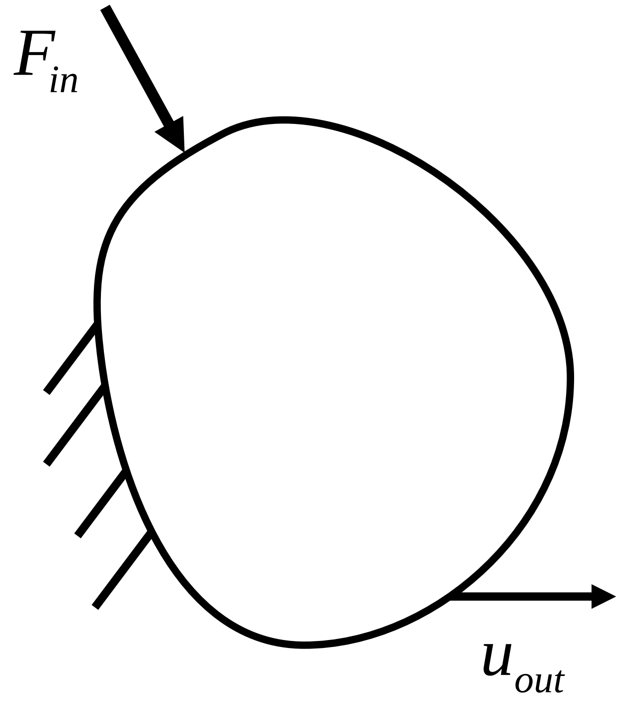
## „Characteristic stiffness formulation“ according to Wang

$$\begin{bmatrix} \mathbf{k}_{aa} & \mathbf{k}_{ac} \\ \mathbf{k}_{ca} & \mathbf{k}_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}_a \\ 0 \end{bmatrix}$$

$$(\mathbf{k}_{aa} - \mathbf{k}_{ac}\mathbf{k}_{cc}^{-1}\mathbf{k}_{ca})\mathbf{u}_a = \bar{\mathbf{k}}\mathbf{u}_a = \mathbf{f}_a$$

$$\begin{bmatrix} F_{in} \\ F_{out} \end{bmatrix} = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} \\ \bar{k}_{21} & \bar{k}_{22} \end{bmatrix} \begin{bmatrix} u_{in} \\ u_{out} \end{bmatrix}$$

$$f(\mathbf{x}) = \mathrm{e}^{-(GA-GA_d)^2} k_{11} k_{22}$$




(a)

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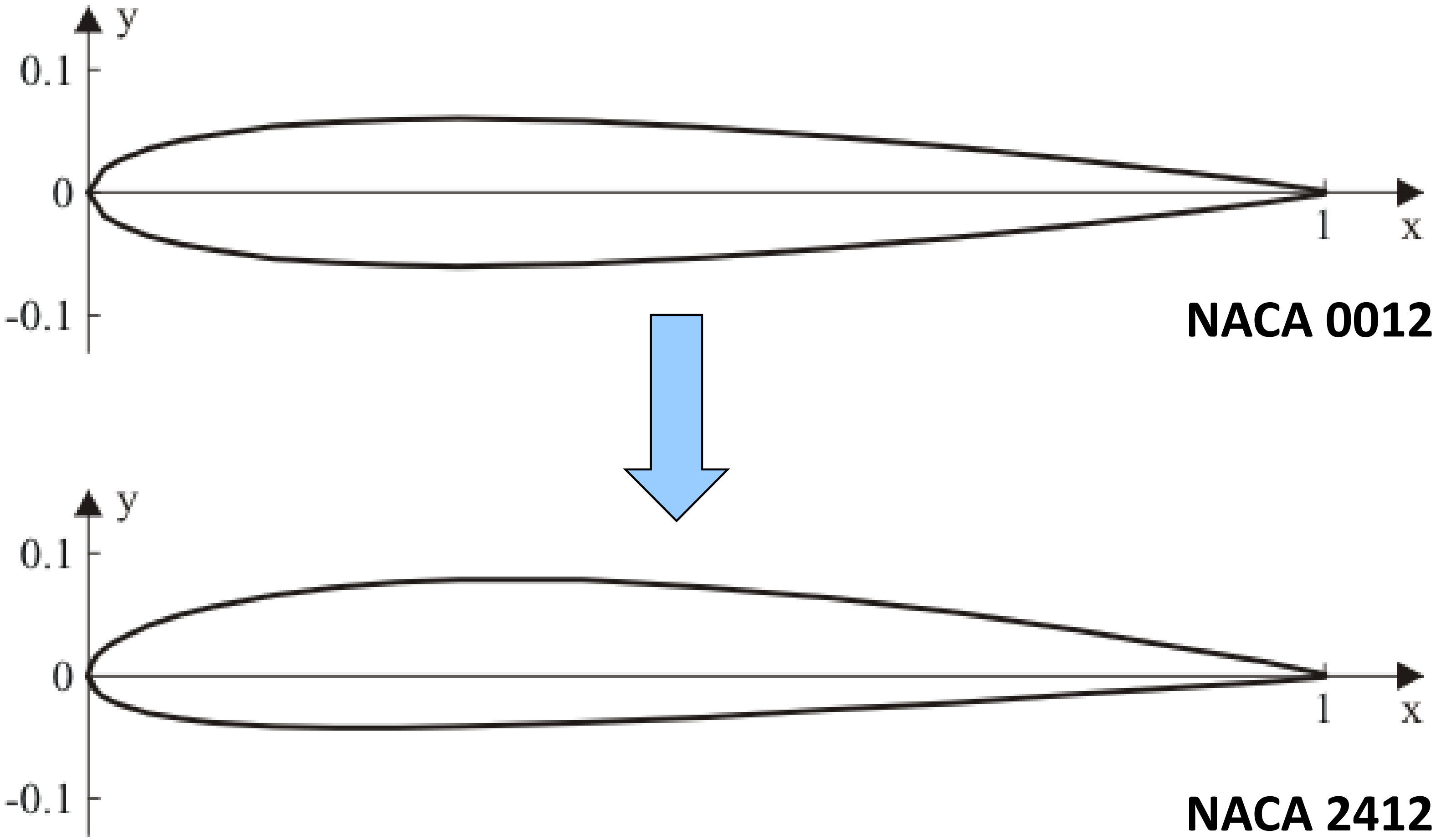
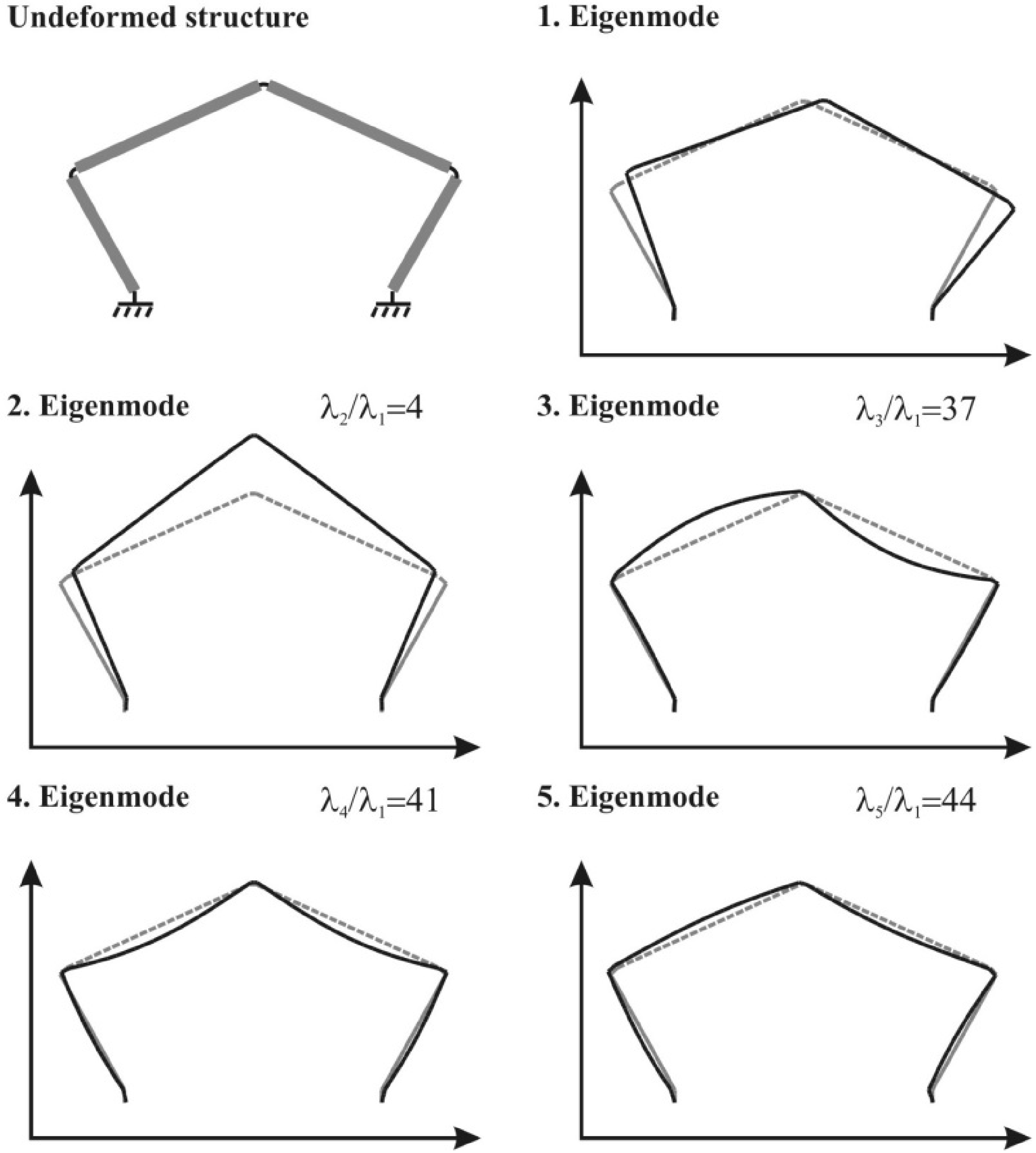
# Formulation for mechanisms with multiple outputs




**NACA 0012**

**NACA 2412**

# Design criterion




$$\mathbf{k}\boldsymbol{\varphi} = \boldsymbol{\lambda}\boldsymbol{\varphi}$$

$$\boldsymbol{\varphi}^T\boldsymbol{\varphi} = 1$$

$$\frac{1}{2}\boldsymbol{\varphi}^T\mathbf{k}\boldsymbol{\varphi} = \frac{1}{2}\boldsymbol{\lambda}\boldsymbol{\varphi}^T\boldsymbol{\varphi} = \frac{1}{2}\boldsymbol{\lambda}$$

$$\mathbf{u} = \alpha\boldsymbol{\varphi}$$

$$SE = \alpha^2\frac{1}{2}\boldsymbol{\varphi}^T\mathbf{k}\boldsymbol{\varphi}$$

$$\frac{1}{2}\boldsymbol{\varphi}^T\mathbf{k}\boldsymbol{\varphi} = \frac{SE}{\alpha^2}$$

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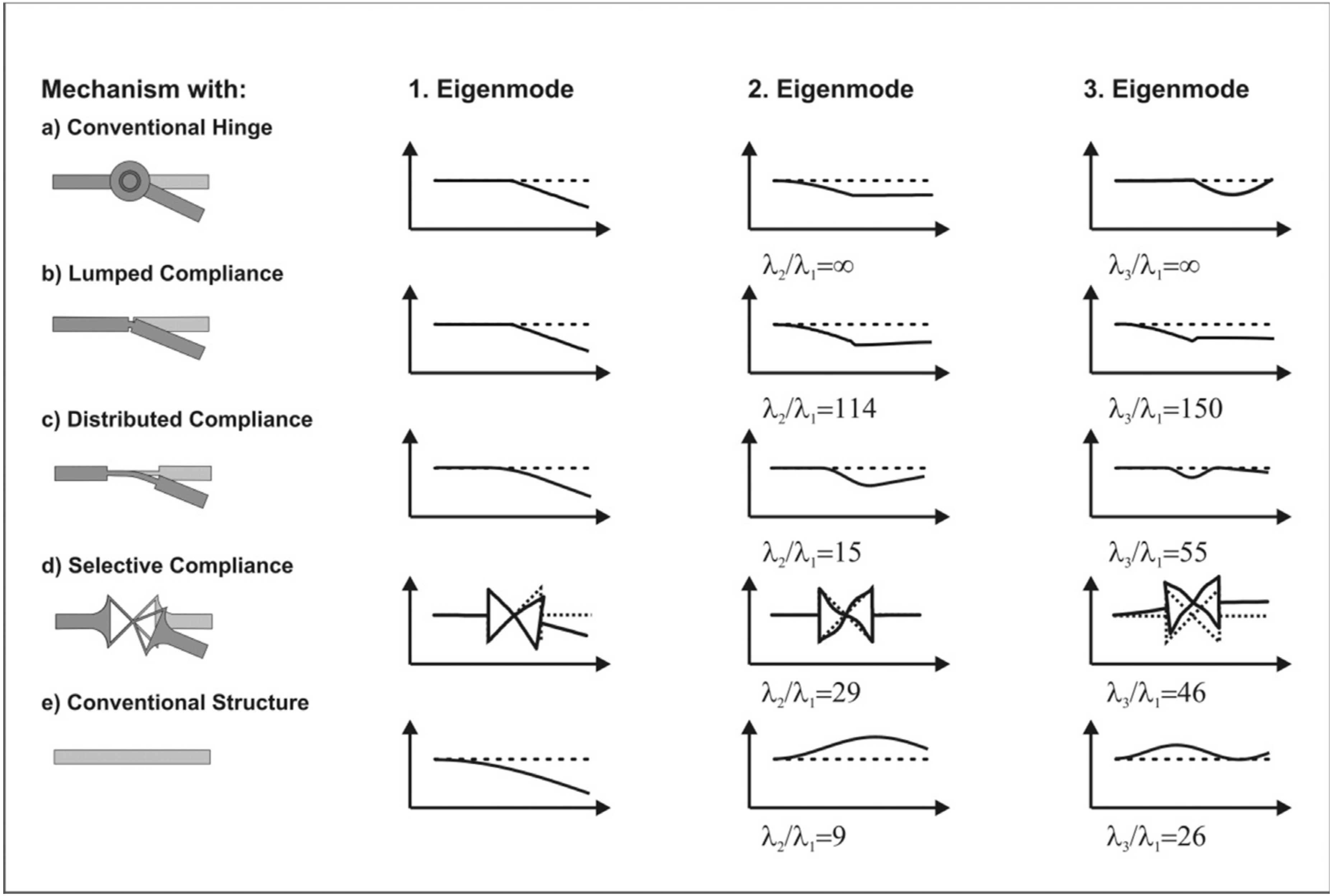
# Design criterion



# Design criterion

$$\frac{1}{2}\boldsymbol{\Phi}^T\mathbf{k}\boldsymbol{\Phi} = \frac{1}{2}\mathbf{K} = \frac{1}{2}\begin{bmatrix} \lambda_1 & 0 & & \cdots & & 0 \\ 0 & \ddots & 0 & & & \\ & 0 & & 0 & & \vdots \\ \vdots & & 0 & & 0 & \\ & & & 0 & \ddots & 0 \\ 0 & & \cdots & & 0 & \lambda_p \end{bmatrix}$$

$$k = \lambda_1$$

$$\bar{k} = \{\lambda_2, \lambda_3, \ldots, \lambda_n\}$$

$$\boldsymbol{\varphi}_1 = \boldsymbol{\chi}_1$$

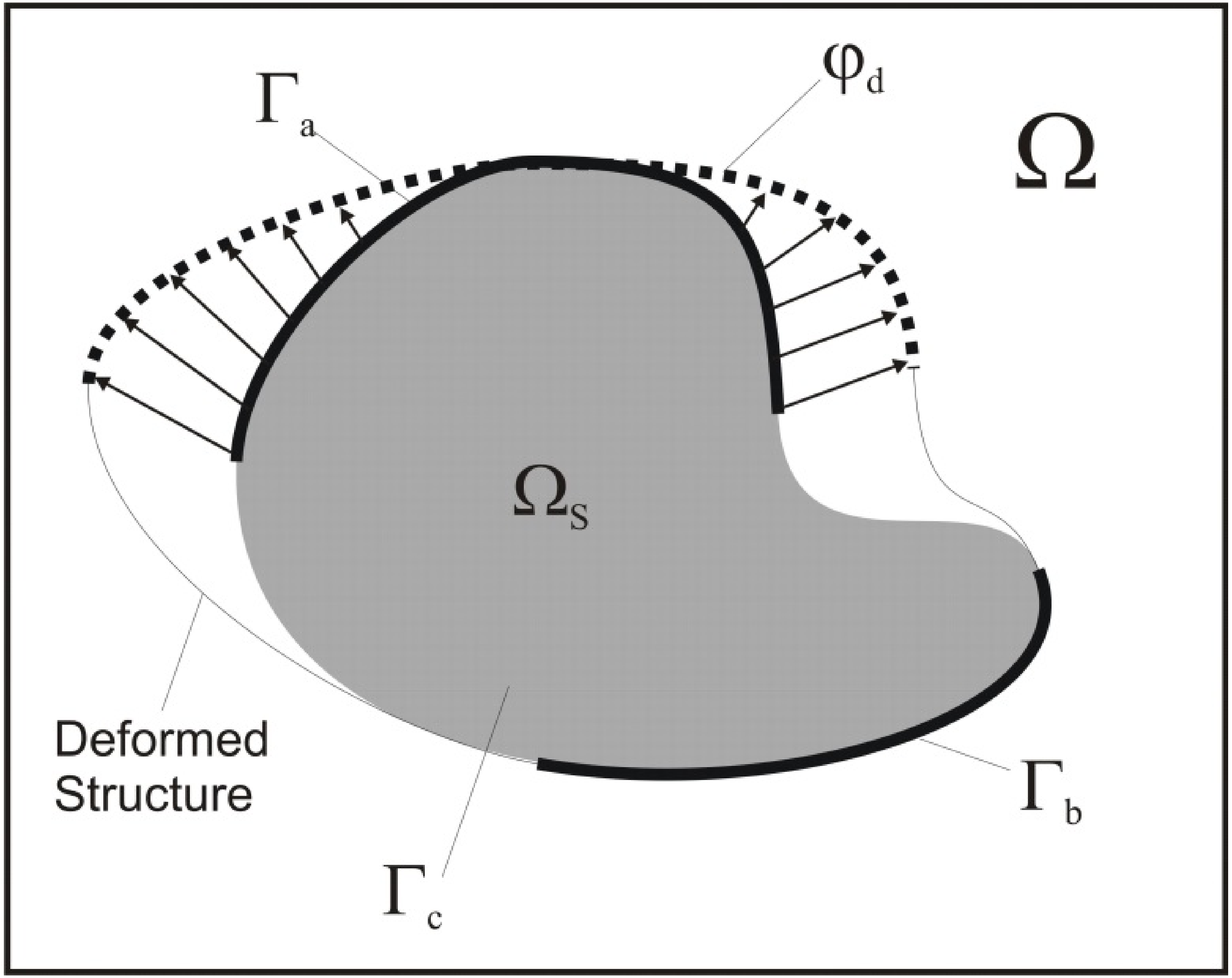
# Design criterion

$$\chi_i^T \chi_i = 1 \quad , \quad \forall i = 1 \ldots m$$
$$\bar{\chi}_i^T \bar{\chi}_i = 1 \quad , \quad \forall i = m+1 \ldots p$$

$$\mathrm{X} = \begin{bmatrix} \chi_1 & \cdots & \chi_m & \bar{\chi}_{m+1} & \cdots & \bar{\chi}_p \end{bmatrix}$$

$$\frac{1}{2}\mathrm{X}^T \mathbf{k} \mathrm{X} = \frac{1}{2}\mathbf{K} = \frac{1}{2}\begin{bmatrix} k_1 & 0 & & \cdots & & 0 \\ 0 & \ddots & & & & \\ & & k_m & \ddots & & \vdots \\ \vdots & & \ddots & \bar{k}_{m+1} & & \\ & & & & \ddots & 0 \\ 0 & & \cdots & & 0 & \bar{k}_p \end{bmatrix}$$

# Optimization formulation

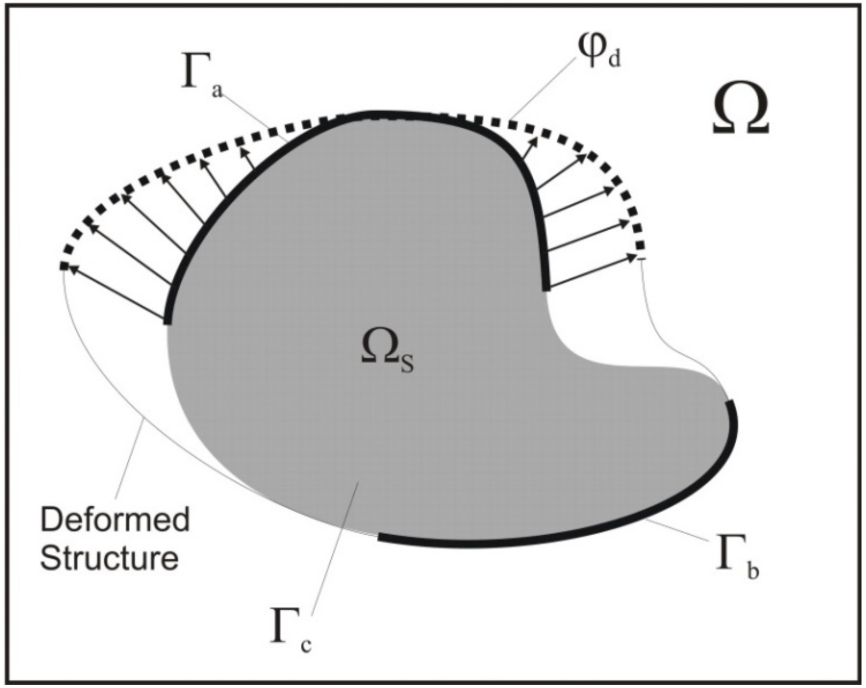


# Optimization formulation

$$\begin{bmatrix} \mathbf{k}_{aa} & \mathbf{k}_{ac} \\ \mathbf{k}_{ca} & \mathbf{k}_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}_a \\ \mathbf{f}_c \end{bmatrix}$$

$$\overline{\mathbf{k}} = (\mathbf{k}_{aa} - \mathbf{k}_{ac}\mathbf{k}_{cc}^{-1}\mathbf{k}_{ca})$$

$$\overline{\mathbf{k}}\overline{\boldsymbol{\varphi}} = \overline{\boldsymbol{\lambda}}\overline{\boldsymbol{\varphi}}$$




Problem

$$\overline{\boldsymbol{\varphi}}_1 \neq \overline{\boldsymbol{\varphi}}_d$$

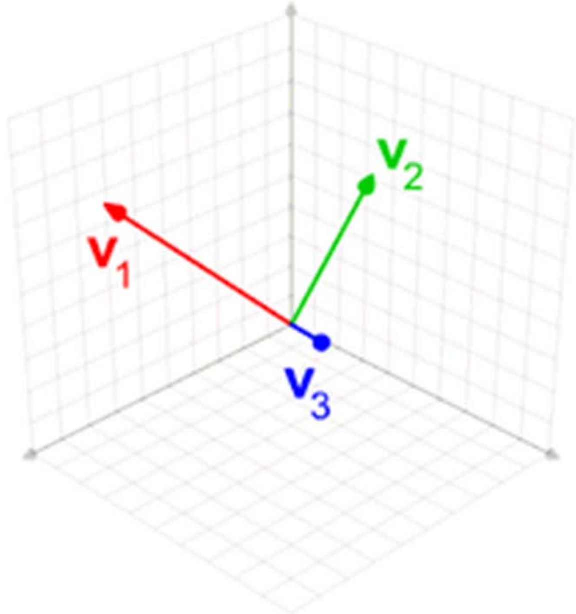
Solution

$$\overline{\mathbf{k}}\overline{\boldsymbol{\psi}} = \tilde{\lambda}\overline{\mathbf{w}}\overline{\boldsymbol{\psi}}$$ mit $$\overline{\boldsymbol{\Psi}}_1 = \boldsymbol{\varphi}_d$$

# Optimization formulation

$$\hat{\mathbf{\Phi}} = \left[ \bar{\boldsymbol{\varphi}}_d \,|\, \bar{\boldsymbol{\varphi}}_1 \,|\, \quad \ldots \quad |\, \bar{\boldsymbol{\varphi}}_{j-1} \,|\, \bar{\boldsymbol{\varphi}}_{j+1} \,|\, \quad \ldots \quad |\, \bar{\boldsymbol{\varphi}}_q \right]$$

with $\left|\bar{\mathbf{a}}_{dj}\right| = \max_{i=1\ldots q} \left|\bar{a}_{di}\right|$




$$\bar{\boldsymbol{\psi}}_1 = \bar{\boldsymbol{\varphi}}_d$$

$$\bar{\boldsymbol{\psi}}_2 = \hat{\boldsymbol{\varphi}}_2 - \frac{\bar{\boldsymbol{\psi}}_1^T \bar{\mathbf{k}} \hat{\boldsymbol{\varphi}}_2}{\bar{\boldsymbol{\psi}}_1^T \bar{\mathbf{k}} \bar{\boldsymbol{\psi}}_1} \bar{\boldsymbol{\psi}}_1$$

$$\bar{\boldsymbol{\psi}}_q = \hat{\boldsymbol{\varphi}}_q - \sum_{i=1}^{q-1} \frac{\bar{\boldsymbol{\psi}}_i^T \bar{\mathbf{k}} \hat{\boldsymbol{\varphi}}_q}{\bar{\boldsymbol{\psi}}_i^T \bar{\mathbf{k}} \bar{\boldsymbol{\psi}}_i} \bar{\boldsymbol{\psi}}_i$$

$$\bar{\mathbf{\Psi}} = \left[ \bar{\boldsymbol{\psi}}_1 \;|\; \bar{\boldsymbol{\psi}}_2 \;|\; \ldots \;|\; \bar{\boldsymbol{\psi}}_q \right]$$

# Optimization formulation

$$\bar{\mathbf{k}}\bar{\boldsymbol{\psi}} = \tilde{\lambda}\bar{\mathbf{w}}\bar{\boldsymbol{\psi}}$$ with $$\bar{\boldsymbol{\Psi}}_1 = \boldsymbol{\varphi}_d$$

$$\bar{\boldsymbol{\Lambda}} = \bar{\boldsymbol{\Psi}}^T \bar{\mathbf{k}} \bar{\boldsymbol{\Psi}} = \begin{bmatrix} \tilde{\lambda}_1 & 0 & & 0 \\ 0 & \ddots & 0 & \\ & 0 & \ddots & 0 \\ 0 & & 0 & \tilde{\lambda}_q \end{bmatrix}$$

Objective function

$$f(\mathbf{x}) = \frac{\tilde{\lambda}_1}{\min\left\{\tilde{\lambda}_{2..q}\right\}}$$

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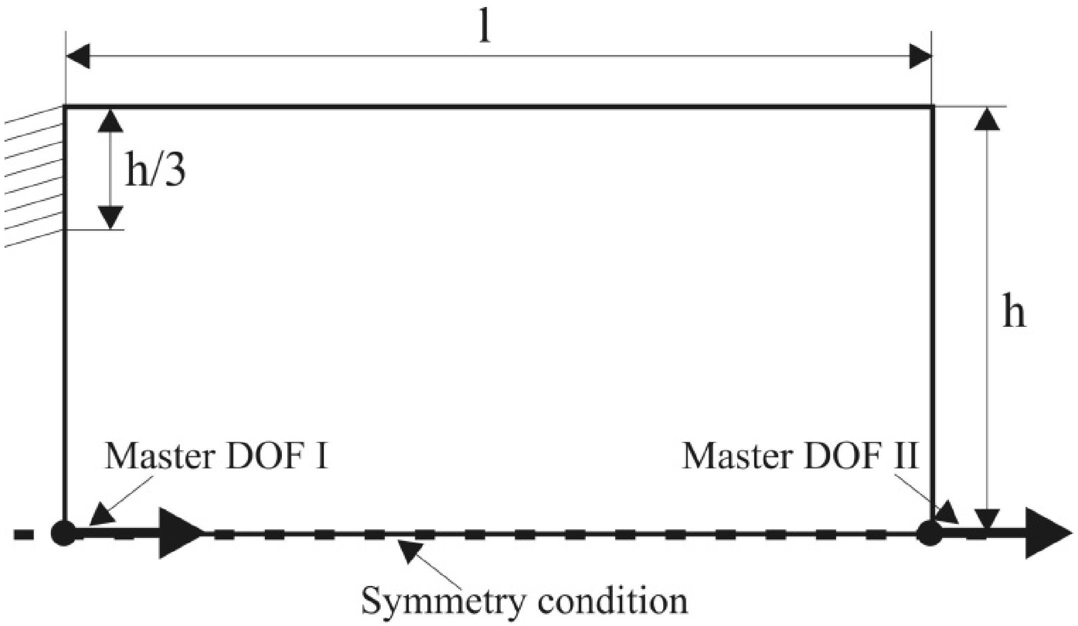
# Design example – force inverter

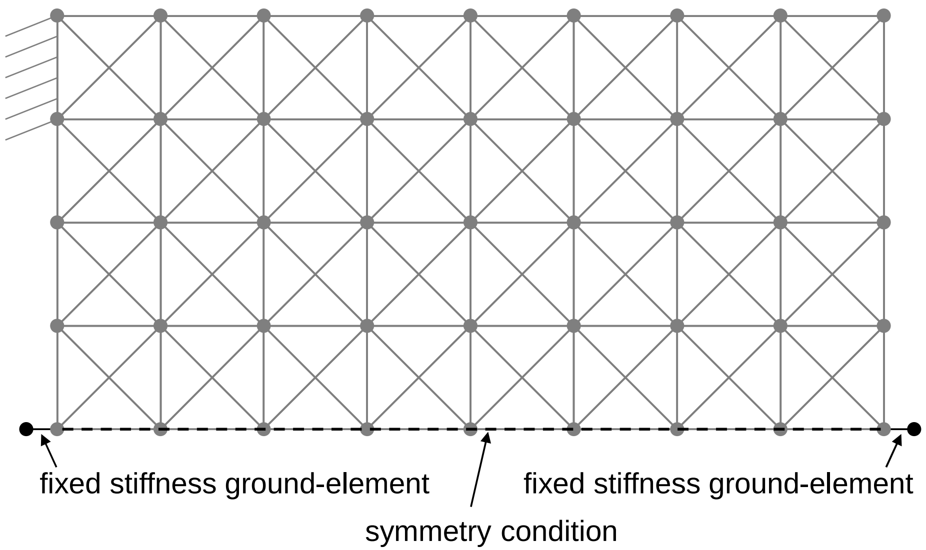
## Problem statement and parameterization




Design domain with support and master DoFs; arrows define the directions of positive displacement in the master DoFs

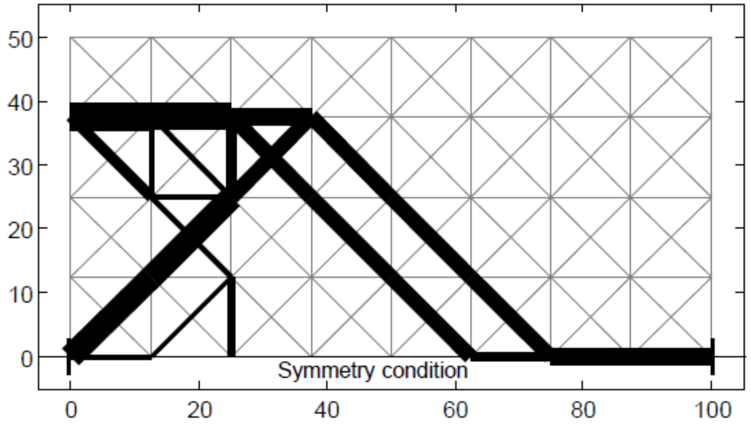
$$\mathbf{k}(\mathbf{x}) = \sum_{i=1}^{n} x_i \mathbf{k}_i$$

Parameterized design domain

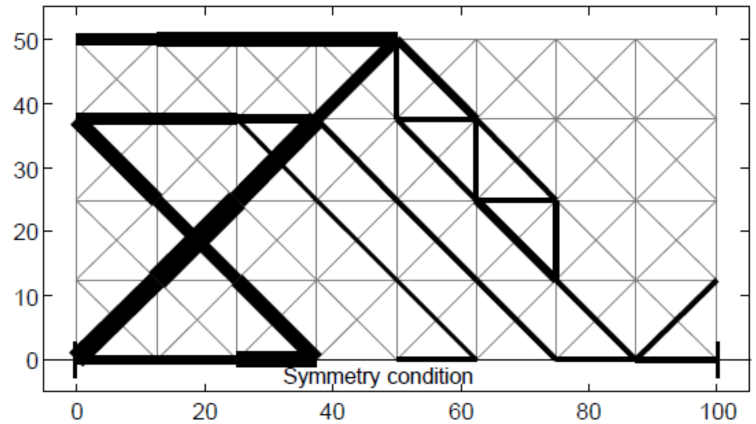


# Design example – force inverter
## Results and discussion

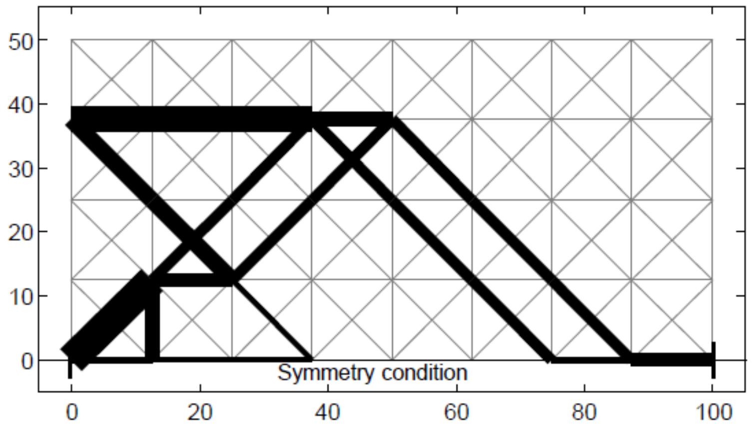
a) $\chi = [1 \quad -1]^T, \ f(\mathbf{x}) = -15.30, \ g(\mathbf{x}) = -2.20 \cdot 10^{-5}$



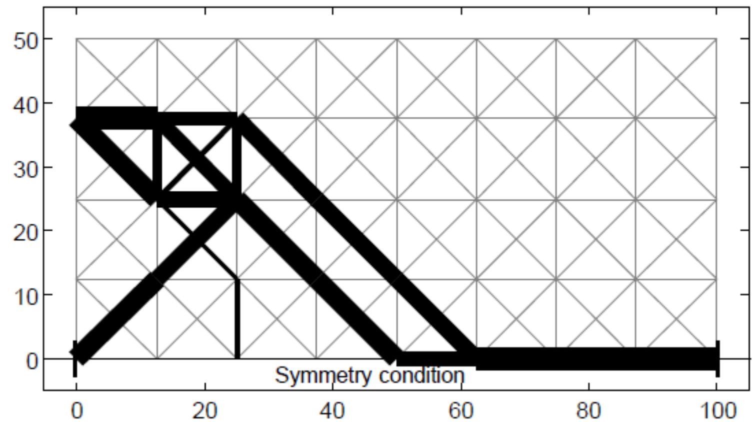

c) $\chi = [1 \quad -3]^T, \ f(\mathbf{x}) = -15.16, \ g(\mathbf{x}) = -0.028$




b) $\chi = [1 \quad -2]^T, \ f(\mathbf{x}) = -15.44, \ g(\mathbf{x}) = -0.008$




d) $\chi = [2 \quad -1]^T, \ f(\mathbf{x}) = -15.36, \ g(\mathbf{x}) = -7 \cdot 10^{-9}$

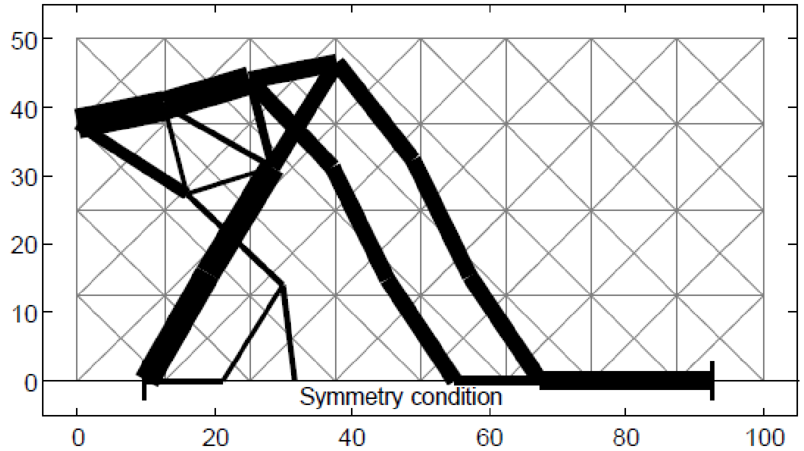


# Design example – force inverter
## Eigenvalue analysis of the force inverters

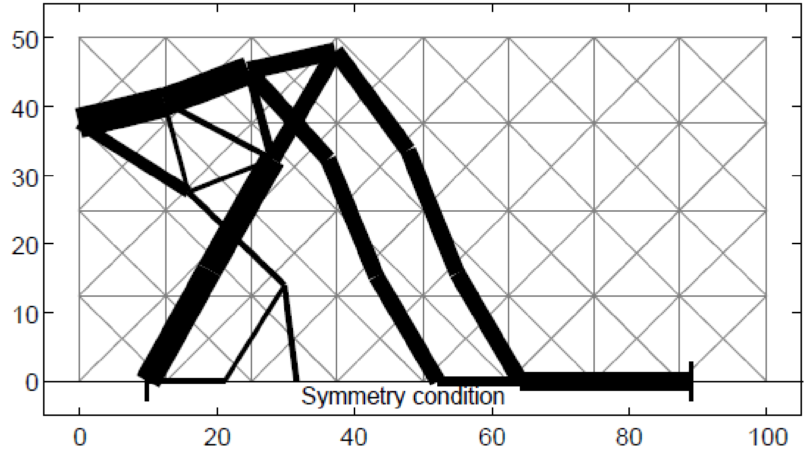
| $\chi$ | $\varphi_1$ | $\varphi_2$ | $\lambda_1$ | $\lambda_2$ |
|---|---|---|---|---|
| $\begin{bmatrix} 0.7071 \\ -0.7071 \end{bmatrix}$ | $\begin{bmatrix} 0.7046 \\ -0.7096 \end{bmatrix}$ | $\begin{bmatrix} 0.7096 \\ 0.7046 \end{bmatrix}$ | 16.9946 | 123.5943 |
| $\begin{bmatrix} 0.4472 \\ -0.8944 \end{bmatrix}$ | $\begin{bmatrix} 0.4490 \\ -0.8944 \end{bmatrix}$ | $\begin{bmatrix} 0.8990 \\ 0.4400 \end{bmatrix}$ | 16.9660 | 133.1973 |
| $\begin{bmatrix} 0.3162 \\ -0.9487 \end{bmatrix}$ | $\begin{bmatrix} 0.2999 \\ -0.9543 \end{bmatrix}$ | $\begin{bmatrix} 0.9543 \\ 0.2989 \end{bmatrix}$ | 16.8673 | 116.0604 |
| $\begin{bmatrix} 0.8944 \\ -0.4472 \end{bmatrix}$ | $\begin{bmatrix} 0.8975 \\ -0.4410 \end{bmatrix}$ | $\begin{bmatrix} 0.4410 \\ 0.8975 \end{bmatrix}$ | 16.9777 | 137.6287 |

# Design example – force inverter
## Deformation plots

a) $f_{in} = 300N$ , $u_{in} = 9.98mm$ , $f_{out} = 0N$ , $u_{out} = -7.61mm$

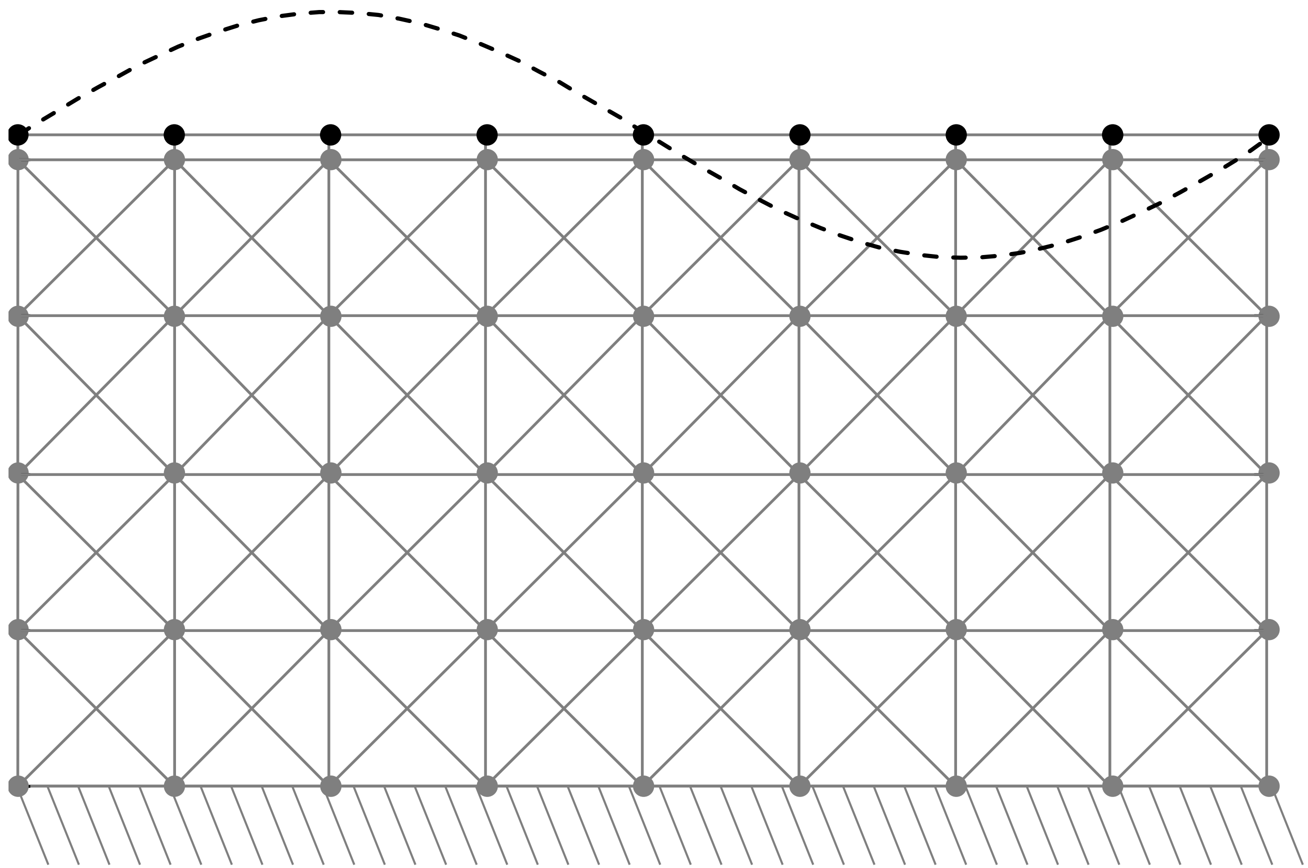



b) $f_{in} = 120N$ , $u_{in} = 10.08mm$ , $f_{out} = -240N$ , $u_{out} = -11.12mm$

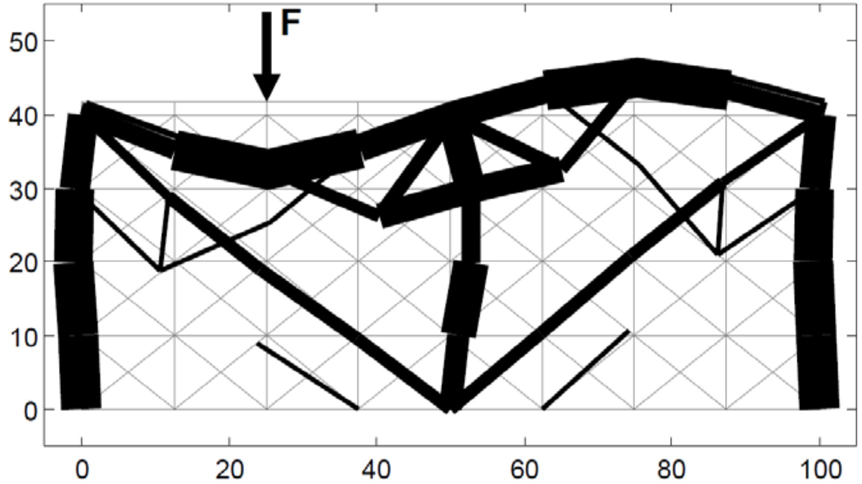


# Design Example – shape adaptive structure

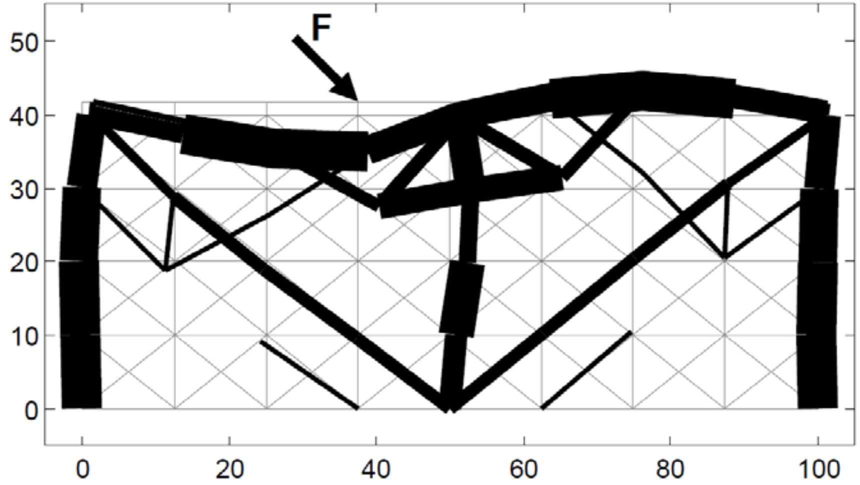
## Problem statement



# Design Example – shape adaptive structure
## Results

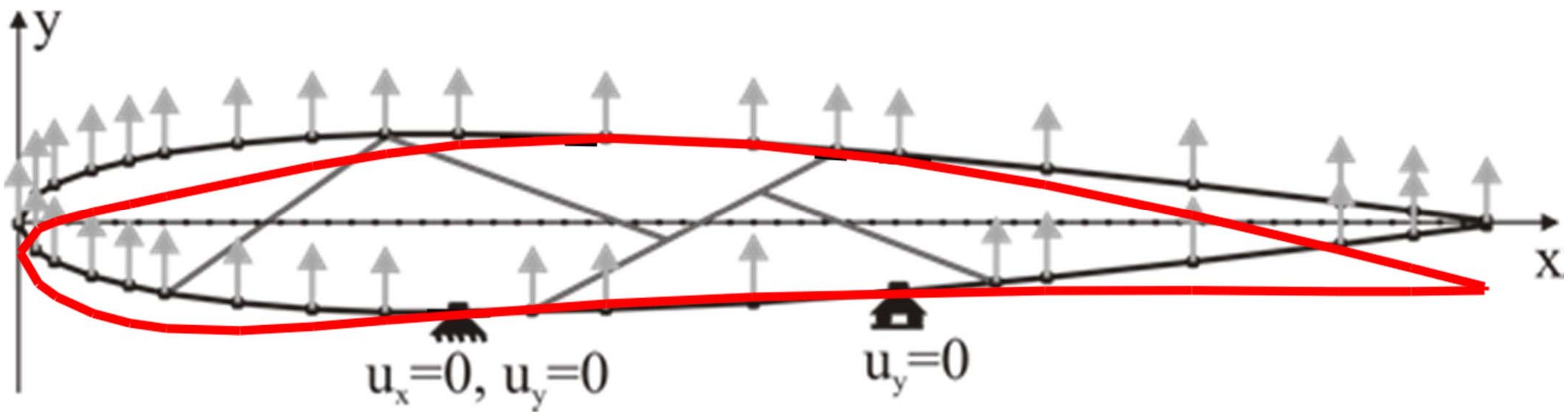



*Load case 1,* F =-140 *N*




*Load case 2,* Fx =140 *N,* Fy =-140 *N*

# Design example – Morphing wing




$$\mathbf{k} = \mathbf{k}_{Belt} + \mathbf{k}_{Stiff}(\mathbf{x})$$

Stiffness matrix

$$\varphi_d$$

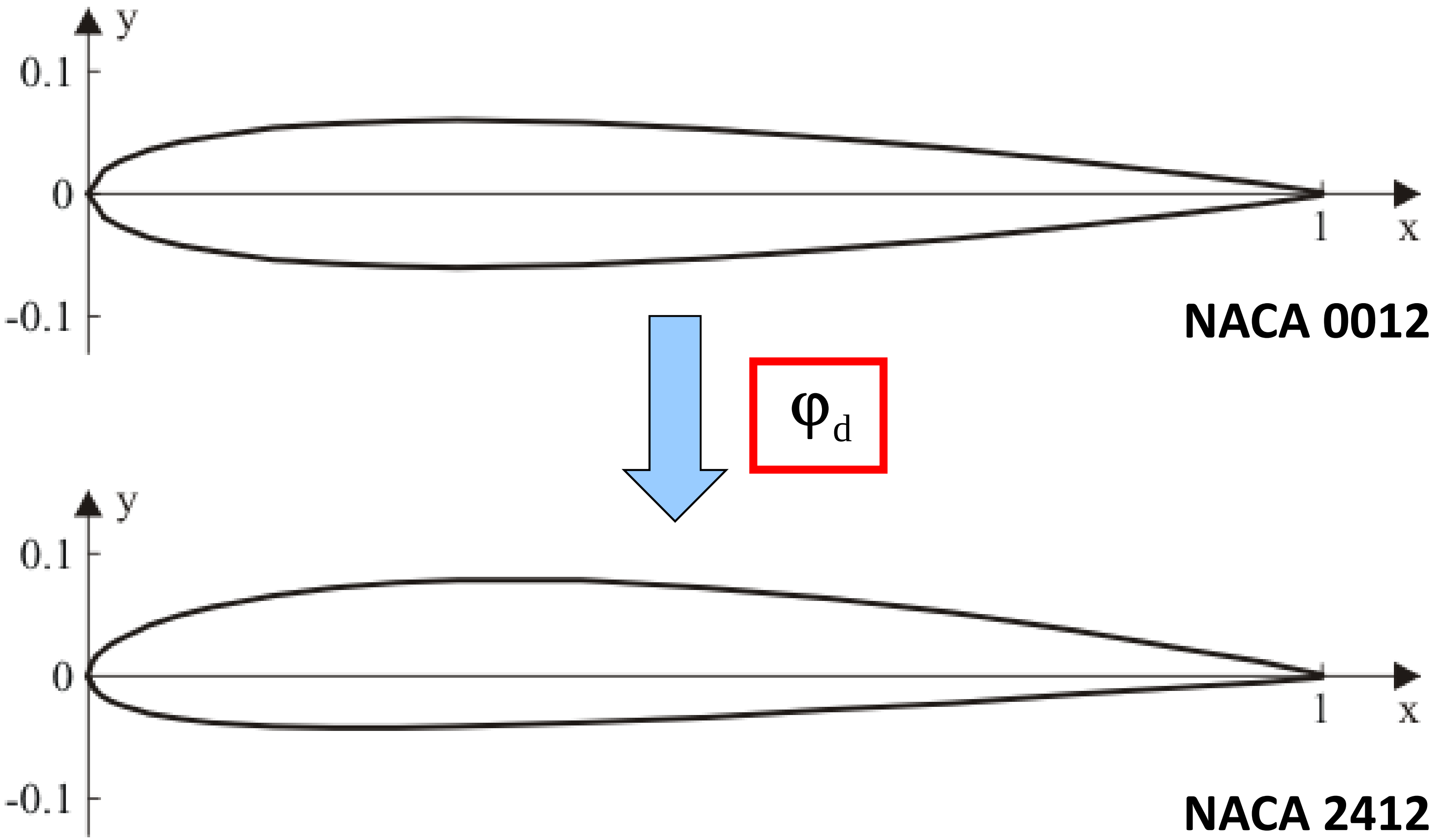
$$\overline{\mathbf{k}}\varphi = \lambda\varphi$$

Eigenvalue problem

$$\varphi_1 \approx \varphi_d$$
$$\lambda_1 \ll \lambda_2$$

Design criterion

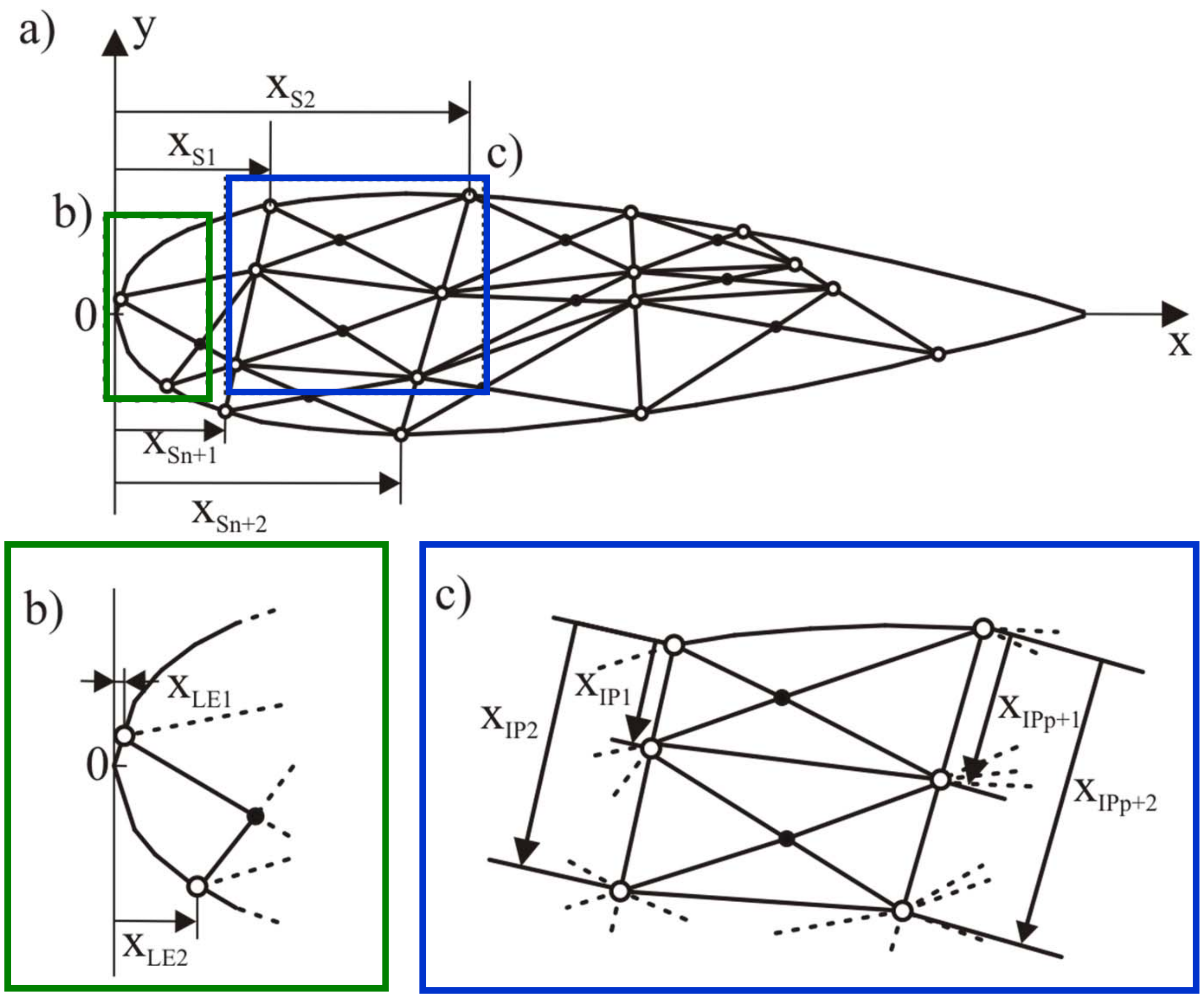
# Design example – Morphing wing



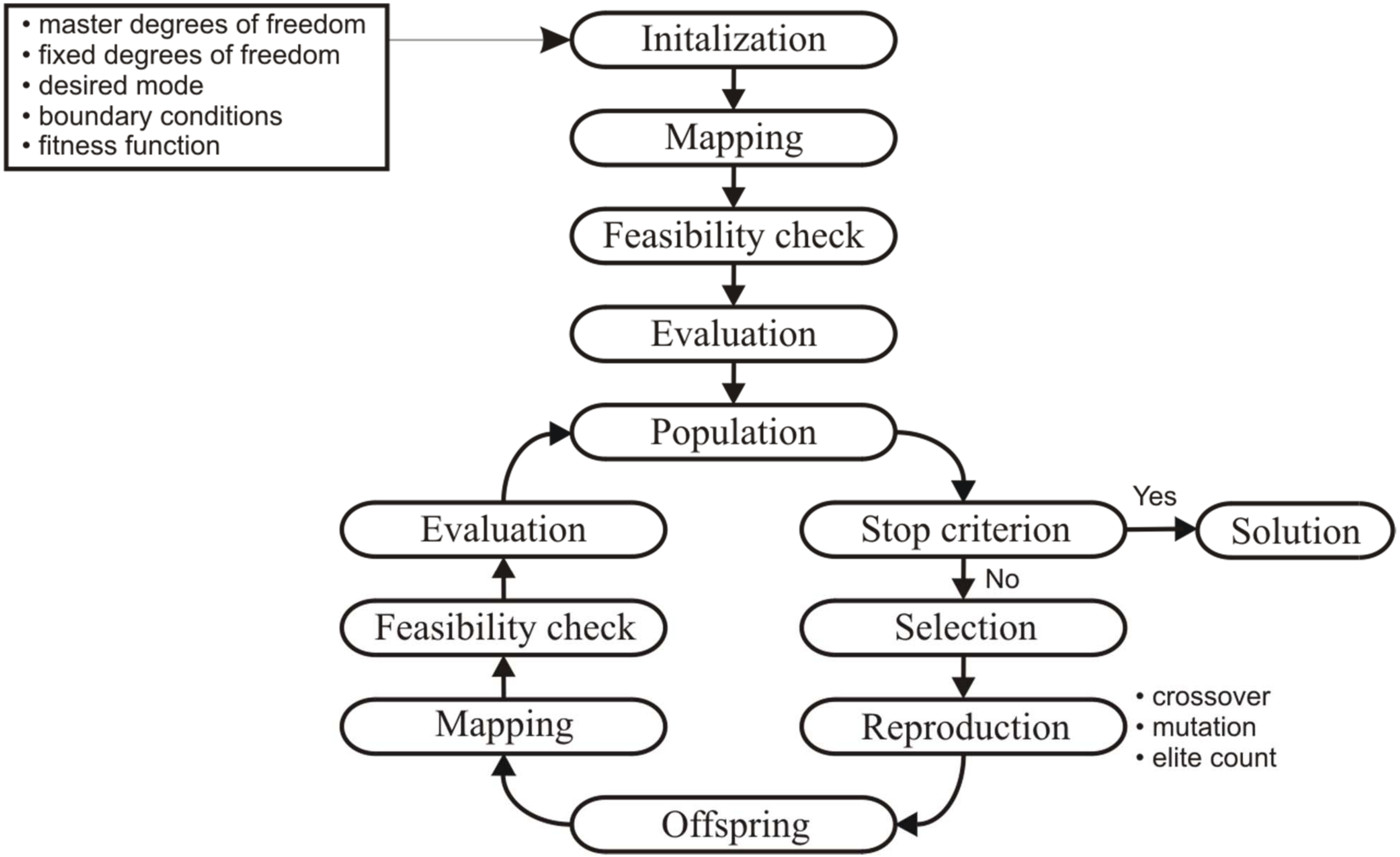
# Design example – Morphing wing

$$\mathbf{k} = \mathbf{k}_{Belt} + \mathbf{k}_{Stiff}(\mathbf{x})$$

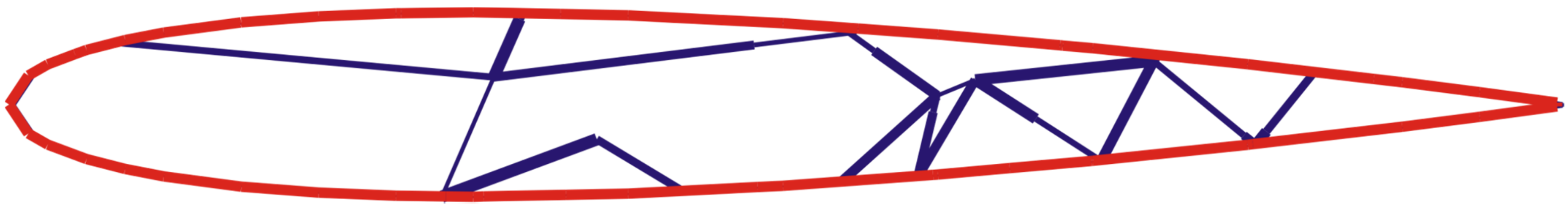
$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_S \\ \mathbf{x}_{IP} \\ \mathbf{x}_{LE} \\ \mathbf{x}_T \end{bmatrix}$$



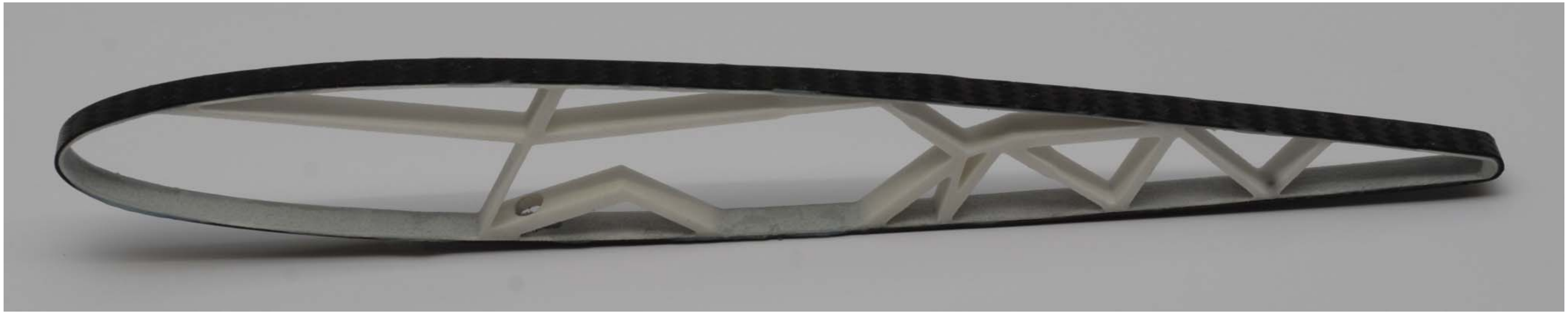
# Design example – Morphing wing
## Optimization procedure



# Design Example – Morphing wing



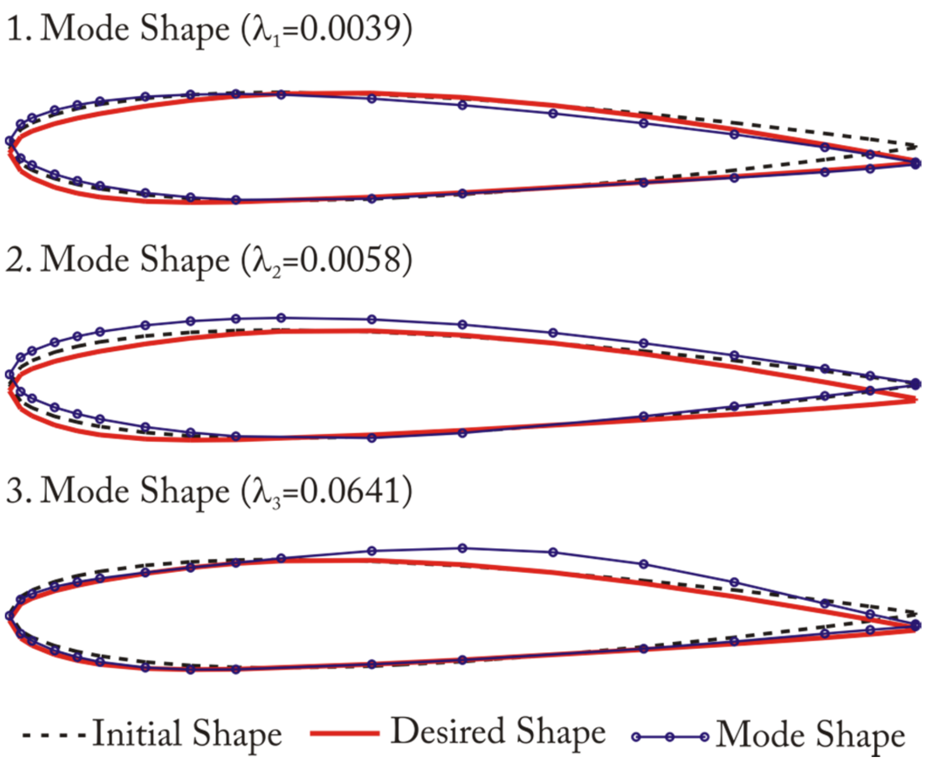
Output from the design routine



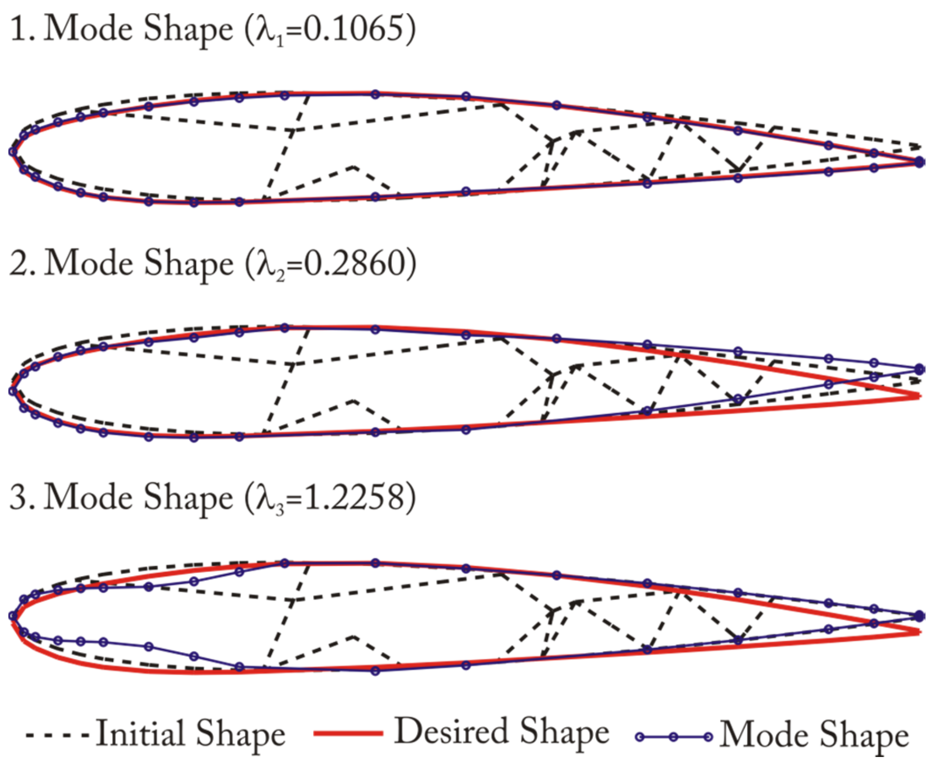
Prototype as hybrid design: CFRP belt and inner stiffening structure in Polyamide

# Design example – Morphing wing

## Single Belt




## Complete Belt-Rib Structure

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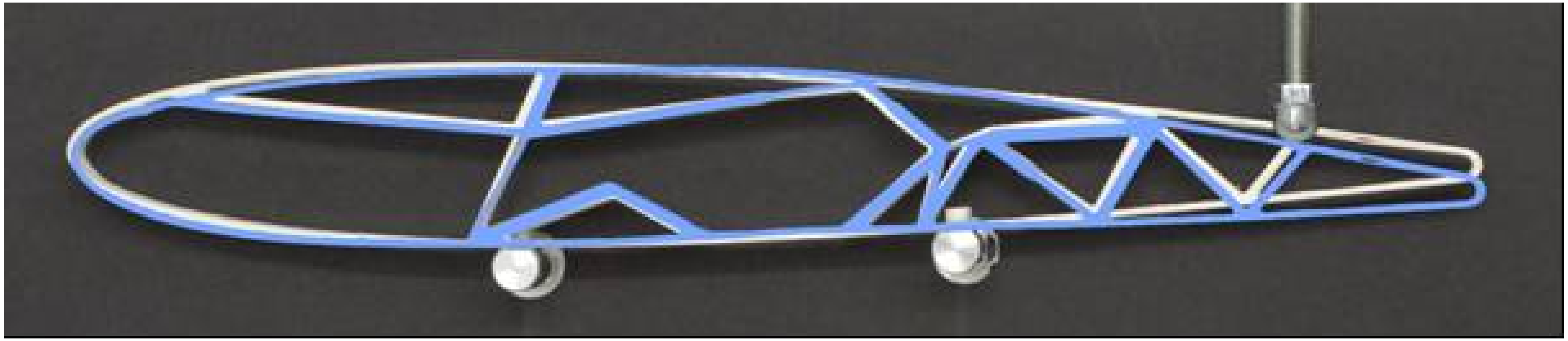
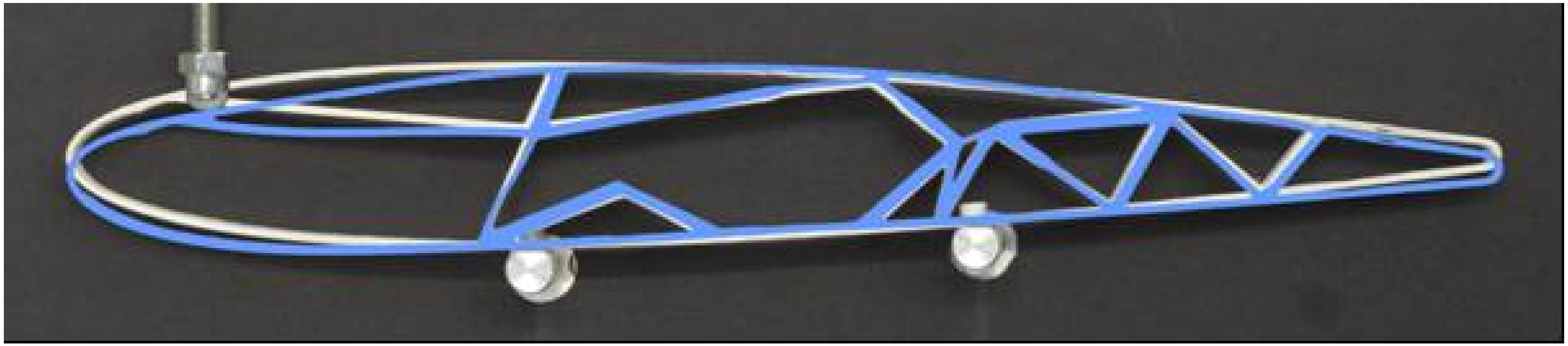
# Experimental investigations

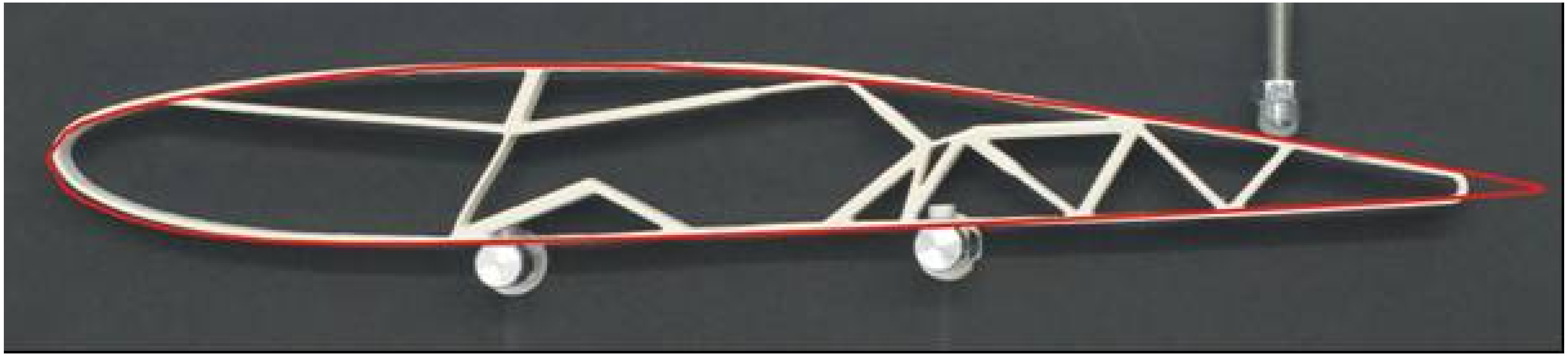


Applied displacement at the trailing edge

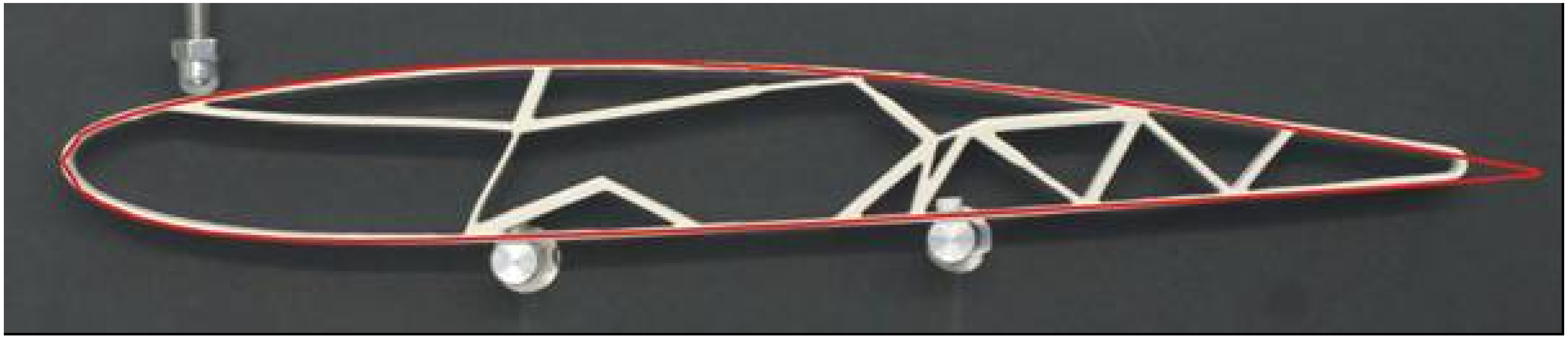


Applied displacement at the leading edge

# Experimental investigations



Comparison between the desired and the actual deformed shapes by an applied displacement of 3.5 mm



Comparison between the desired and the actual deformed shapes by an applied displacement of 3.5 mm

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Prof. Dr.-Ing. Sandro Wartzack

# Thank you for your attention.