How Often is the Conclusion of Euclid's Lemma True?

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Euclid's Lemma is one of the oldest results in number theory, and it describes a fundamental property of prime numbers. The lemma says that if a product of two positive integers is divisible by a prime number, then one of the two integers must itself be divisible by the prime. A recent paper by Adrian Dudek calculates the probability of the lemma being true when applied to three randomly chosen positive integers. Specifically, Dudek finds that the probability is asymptotic to $\pi^2/\log(N)$, where N is the upper bound on the three integers. We consider a similar but more general question; how often does the lemma hold if the product of two randomly chosen positive integers is replaced by a product of an arbitrary list of numbers?